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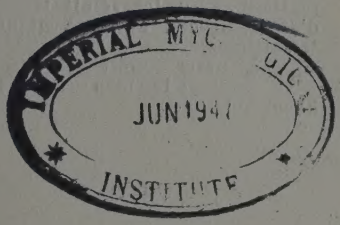

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MATHÉMATIQUES

REPRÉSENTATION GÉNÉRALE DES SOLUTIONS D'UNE ÉQUATION DIFFÉRENTIELLE DES FONCTIONS SPHÉRIQUES

Par I. N. VEKOUA

(Présenté par N. I. Mouskhelichvili, de l'Académie, le 7. IV. 1945)

Le but de la présente Note est d'exprimer au moyen des fonctions analytiques d'une variable complexe toutes les solutions de l'équation différentielle

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial u}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2 u}{\partial\varphi^2}+n(n+1)u=0 \qquad (1)$$

où θ, φ sont les coordonnées géographiques d'un point sur la sphère ($0\leqslant\theta\leqslant\pi$, $0\leqslant\varphi<2\pi$) et n est une constante en général complexe.

L'équation (1) peut être écrite dans la forme

$$\frac{\partial^2 u}{\partial z\partial\zeta}+\frac{n(n+1)}{(1+z\zeta)^2}u=0 \qquad (2)$$

où

$$z=\operatorname{tg}\frac{\theta}{2}e^{i\varphi}, \qquad \zeta=\operatorname{tg}\frac{\theta}{2}e^{-i\varphi} \qquad (3)$$

Donc, en vertu de la formule générale que j'ai obtenue dans mes articles([1],[2]), toutes les solutions de l'équation (1) peuvent au voisinage de $\theta=0$ être représentées sous la forme

$$u=\varphi(z)+\int_0^z\beta(z,\zeta,t)\varphi(t)\,dt+\psi(\zeta)+\int_0^\zeta\beta'(z,\zeta,\xi)\psi(\xi)\,d\xi \qquad (4)$$

où φ, ψ sont des fonctions arbitraires analytiques au voisinage de l'origine des coordonnées, et β et β' ont la forme

$$\beta(z,\zeta,t)=\int_0^\xi V(z,\zeta,t,\xi)\,d\xi, \qquad \beta'(z,\zeta,\xi)=\int_0^z V(z,\zeta,t,\xi)\,dt \qquad (5)$$

La fonction V, qui dépend de 4 arguments complexes z, ζ, t, ξ, vérifie par rapport aux deux premiers arguments l'équation (2) et d'ailleurs les conditions

$$V(t,\zeta,t,\xi)=V(z,\xi,t,\xi)=-\frac{n(n+1)}{(1+\xi,^2} \qquad (6)$$

qui la déterminent complètement.

Démontrons maintenant que

$$V(z, \zeta, t, \xi) = -\frac{n(n+1)}{(1+\xi)^2} P_n(\omega) \tag{7}$$

où P_n est fonction de Legendre de première espèce (3), et

$$\omega(z, \zeta, t, \xi) = \frac{(1-z\zeta)(1-t\xi)+2z\xi+2\zeta t}{(1+z\zeta)(1+t\xi)} \tag{8}$$

Nous avons

$$\left.\begin{aligned} &\frac{\partial\omega}{\partial z} = \frac{2(\xi-\zeta)(1+\zeta t)}{(1+z\zeta)^2(1+t\xi)}, \quad \frac{\partial\omega}{\partial\zeta} = \frac{2(t-z)(1+z\xi)}{(1+z\zeta)^2(1+t\xi)} \\ &\frac{\partial^2\omega}{\partial z\partial\zeta} = -\frac{2\omega}{(1+z\zeta)^2}, \quad \frac{\partial\omega}{\partial z}\frac{\partial\omega}{\partial\zeta} = \frac{1-\omega^2}{(1+z\zeta)^2} \end{aligned}\right\} \tag{9}$$

En vertu des formules (9) nous obtenons immédiatement

$$\frac{\partial^2 P_n(\omega)}{\partial z\partial\zeta} + \frac{n(n+1)}{(1+z\zeta)^2} P_n(\omega) \equiv$$

$$\equiv \frac{1}{(1+z\zeta)^2}\left[(1-\omega^2)P_n''(\omega) - 2\omega P_n'(\omega) + n(n+1)P_n(\omega)\right] = 0 \tag{10}$$

ce qui démontre que l'expression (7) vérifie l'équation (2). Ensuite, en tenant compte de ce que

$$\omega(t, \zeta, t, \xi) = \omega(z, \xi, t, \xi) = 1, \quad P_n(1) = 1$$

nous voyons immédiatement que l'expression (7) vérifie aussi les conditions (6).

Déterminons maintenant β et β'. Comme $\omega(z, \zeta, t, \xi) = \omega(t, \xi, z, \zeta)$, il résulte de (10) que l'on a

$$\frac{\partial^2 P_n(\omega)}{\partial t\,\partial\xi} + \frac{n(n+1)}{(1+t\xi)^2} P_n(\omega) = 0$$

En intégrant cette égalité par rapport à ξ et en tenant compte de (5), (7) et (8) nous obtenons facilement

$$\beta(z, \zeta, t) = -n(n+1)\int_0^\zeta \frac{P_n(\omega)\,d\xi}{(1+\xi)^2} = -\frac{\partial}{\partial}\,P_n\left(\frac{1-z\zeta+2\zeta t}{1+z\zeta}\right) \tag{11}$$

D'une manière analogue on obtient

$$\beta'(z, \zeta, \xi) = -n(n+1)\int_0^z \frac{P_n(\omega)\,dt}{(1+t\xi)^2} = -\frac{\partial}{\partial\xi}\,P_n\left(\frac{1-z\zeta+2z\xi}{1+z\zeta}\right) \tag{12}$$

En vertu de (11) et (12) la formule (4) prend la forme

$$u = \varphi(z) - \int_0^z \varphi(t)\frac{\partial}{\partial t} P_n\left(\frac{1-z\zeta+2\zeta t}{1+z\zeta}\right) dt + \psi(\zeta) - \int_0^\zeta \psi(\xi)\frac{\partial}{\partial\xi} P_n\left(\frac{1-z\zeta+2z\xi}{1+z\zeta}\right) d\xi \tag{13}$$

En intégrant par parties on peut transformer cette formule et lui donner la forme

$$u(\theta, \varphi) = a_0 P_n(\cos\theta) + \int_0^z \Phi(t)\,P_n(\cos\theta + te^{-i\varphi}\sin\theta)\,dt +$$

$$+ \int_0^\zeta \Psi(\xi)\,P_n(\cos\theta + \xi e^{i\varphi}\sin\theta)\,d\xi \tag{14}$$

où $a_0 = \varphi(0) + \psi(0)$, $\Phi(z) = \varphi'(z)$, $\Psi(\zeta) = \psi'(\zeta)$.

Il est évident que a_0 est une constante arbitraire et Φ et Ψ des fonctions analytiques arbitraires.

Nous avons déjà remarqué que la formule (14) donne toutes les solutions de l'équation (1) au voisinage du point $\theta = 0$. Mais il est facile de montrer que cette formule présente toutes les solutions de l'équation (1) dans un domaine arbitraire T simplement connexe situé sur la sphère, contenant le point $\theta = 0$ et ne contenant pas le point $\theta = \pi$. A cet effet, il faut, dans le cas général, considérer les valeurs de P_n sur la surface de Riemann correspondante. Remarquons encore que la formule (14) subsiste dans le cas d'un domaine multiplement connexe sur la sphère (4); dans ce dernier cas les fonctions Φ et Ψ sont, en général, multiformes.

Il faut encore signaler que si le domaine T est étoilé par rapport au point $\theta = 0$, la formule (14) peut être représentée sous la forme

$$u(\theta, \varphi) = a_0 P_n(\cos\theta) + \int_0^1 [z\Phi(zt) + \zeta\Psi(\zeta t)]\, P_n(t + (1-t)\cos\theta)\, dt \qquad (15)$$

Les formules obtenues peuvent encore avoir des applications différentes dans la théorie générale des fonctions sphériques et dans certains problèmes concrets de la physique mathématique. Indiquons quelques uns de ces problèmes.

En faisant la substitution $\varphi = 1/2 \lg z$, $\psi = 1/2 \lg \zeta$ dans la formule (13), nous aurons

$$Q_n(\cos\theta) = P_n(\cos\theta) \lg \operatorname{tg} \frac{\theta}{2} - \int_0^1 \lg t \, \frac{\partial}{\partial t} P_n(t + (1-t)\cos\theta)\, dt \qquad (16)$$

C'est ce que l'on appelle la solution élémentaire de l'équation (1). Il est facile de voir que Q_n est la fonction de Legendre de seconde espèce. Au moyen de cette fonction on peut obtenir des potentiels différents pour la résolution des problèmes limites liés à l'équation (1).

En substituant $a_0 = 0$, $\Psi = 0$, $\Phi = z^{k-1}$ $(k = 1, 2, \ldots)$ dans (14), nous aurons

$$u_k(\theta, \varphi) = e^{ik\varphi} \theta_{n,k}(\cos\theta) \qquad (17)$$

où

$$\theta_{n,k}(\cos\theta) = \left(\operatorname{tg} \frac{\theta}{2}\right)^k \int_0^1 t^{k-1} P_n(t + (1-t)\cos\theta)\, dt \qquad (18)$$

Il est facile de voir que $\theta_{n,k}$ ne diffère que d'un facteur constant de la fonction adjointe de Legendre de première espèce $P_{n,k}$. Donc, la formule (18) peut être prise pour définition des fonctions adjointes de Legendre de première espèce. Il est d'ailleurs facile d'obtenir de (15) les fonctions adjointes de Legendre de seconde espèce.

En substituant dans (15) les séries de Taylor

$$\Phi(z) = \sum_{k=1}^{\infty} (a_k - ib_k) z^{k-1}, \qquad \Psi(\zeta) = \sum_{k=1}^{\infty} (a_k + ib_k) \zeta^{k-1} \qquad (19)$$

dont le rayon de convergence commun est un certain nombre positif ρ, nous aurons en vertu de (16) une série

$$u(\theta, \varphi) = a_0 P_n(\cos\theta) + 2 \sum_{k=1}^{\infty} (a_k \cos k\varphi + b_k \sin k\varphi)\, \theta_{n,k}(\cos\theta) \qquad (20)$$

qui converge uniformément à l'intérieur du segment sphérique $0 \leqslant \theta \leqslant \theta_0$, où $\operatorname{tg} \frac{\theta_0}{2} = \rho$, $\theta_0 \leqslant \pi$.

Il est maintenant facile de démontrer le théorème:

Chaque solution de l'équation (1), *régulière à l'intérieur du segment sphérique* $0 \leqslant \theta \leqslant \theta_0$, $\theta_0 \leqslant \pi$, *peut être développée en une série de la forme* (20) *qui converge uniformément à l'intérieur du segment indiqué.*

On a aussi le théorème (5):

Chaque solution de l'équation (1), *régulière dans un domaine T simplement connexe (sur la sphère) contenant le point* $\theta = 0$ *et ne contenant pas le point* $\theta = \pi$, *peut être approchée au moyen des formes linéaires en fonctions*

$$P_n(\cos\theta), \quad \theta_{n,k}(\cos\theta)\begin{matrix}\cos k\varphi \\ \sin k\varphi\end{matrix}, \quad k = 1, 2, \ldots \tag{21}$$

Indiquons enfin brièvement quelques applications des formules obtenues à la théorie des enveloppes sphériques élastiques. En suivant Goldenveiser (6) et Vlasov (7), les équations d'équilibre des enveloppes sphériques minces et épaisses peuvent être réduites à un système d'équations de la forme (1). En écrivant la solution générale de chacune des équations de ce système sous la forme (13), (14) ou bien (15) précédemment indiquées, nous pourrons finalement trouver pour les déplacements, les efforts et les moments d'une enveloppe sphérique des expressions explicites au moyen des fonctions analytiques d'une seule variable complexe. Je consacrerai un article spécial à l'application de ces formules pour la résolution des problèmes limites de la théorie des enveloppes sphériques.

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Tbilissi.

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MATHEMATICS

ON THE MULTIGROUPS OF COMPLETE CONJUGATE SETS OF ELEMENTS OF A GROUP

By A. P. DIETZMAN

(Communicated by O. J. Schmidt, Member of the Academy, 18. XII. 1944)

It is known that the complete conjugate sets of elements of a group form a commutative multigroup. It is not known, however, to what extent the structure of the multigroup of complete conjugate sets of elements of a group determines the structure of the group itself and, in particular, it is not decided whether or no there exist non-isomorphic groups possessing isomorphic multigroups of complete conjugate sets of elements.

In the present note we consider groups, for which the multigroup formed by the complete conjugate sets of elements is a group or an ultragroup. It is shown that a group is Abelian if and only if the multigroup formed by the complete conjugate sets of elements of this group is a group. Further it is established that a group is solvable if and only if the multigroup formed by its complete conjugate sets of elements is an ultragroup. We establish, besides, a one-to-one correspondence between the normal divisors of a group G and the submultigroups containing the unit of the multigroup Z of complete conjugate sets of elements of the considered group. It is shown that the multigroup of complete conjugate sets of elements of the factor-group G/H is isomorphic to the factor-multigroup $Z/\overline{Z}$, where $\overline{Z}$ is the submultigroup of Z corresponding to the normal divisor H of the group G.

Following M. Drescher and O. Ore [1], we shall call a set of elements Z a multigroup, if in Z is defined an operation of multiplication satisfying the following conditions:

I. The product of two arbitrary elements of Z is a subset of the elements of Z. Under the product AB of two subsets $A=\{a_\alpha\}$ and $B=\{b_\beta\}$ of elements of Z is understood the subset of Z consisting of the elements of all possible products

$$a_\alpha b_\beta; \quad a_\alpha \subset A; \quad b_\beta \subset B$$

II. The multiplication is associative: for any three a, b and c from Z

$$(ab)c = a(bc) = abc$$

III. For any two elements a and b from Z there exist in Z such elements x and y that

$$ax \supset b; \quad ya \supset b$$

In the sequel we shall consider only multigroups containing a unit e, *i. e.* we shall suppose the following condition to be satisfied:

IV. In Z there exists an element e called the unit, such that for every element a from Z

$$ea \supset a; \quad ae \supset a$$

A multigroup is said to be commutative, if for any two elements a and b from Z

$$ab = ba$$

In the sequel a subset $\overline{Z}$ of elements from Z containing the unit of Z and satisfying the conditions I—III stated above with respect to the operation of multiplication defined in Z shall be called a submultigroup of Z.

A multigroup Z is said to be an ultragroup if it is possible to find in Z a finite series of submultigroups

$$Z = Z_0 \supset Z_1 \supset \ldots \supset Z_m = e,$$

where e is the unit of Z, such that the factor-multigroups (1) Z_{i-1}/Z_i, $i = 1, 2, \ldots, m$ are groups.

Denote by $\{C_\alpha\}$, where α is an index belonging to a set M, the set of complete conjugate sets of elements of the group G. It is evident that the conjugate sets $\{C_\alpha\}$, where $\alpha \subset M$, form a commutative multigroup Z containing a unit, if the operation of multiplication is defined to be multiplication of conjugate sets as complexes of elements of the group G.

Theorem 1. *The multigroup formed by the complete conjugate sets of elements of a group G is a group if and only if G is Abelian.*

In fact, let the complete conjugate sets of elements $\{C_\alpha\}$, $\alpha \subset M$, of the group G, considered as the elements of the multigroup Z, form a group.

Denote by $C_1 = e$ the unit of the group G and, consequently, also the unit of the group Z. For every complete conjugate set C_α which is an element of the group Z we may find in Z unique inverse element $C_{\alpha'}$, where α and α' belong to M, such that

$$C_\alpha C_{\alpha'} = C_{\alpha'} C_\alpha = C_1 \tag{1}$$

Suppose that in the group G there exists a conjugate set C_α, $\alpha \subset M$, containing two elements a and b of the group G. Denote by a_1 the element of the group G belonging to the inverse conjugate set $C_{\alpha'}$. Sence $C_1 = e$, from (1) we have

$$a_1 a = e, \quad a_1 b = e$$

Consequently,

$$a = b$$

Thus, we conclude that every complete conjugate sets of elements C_α, $\alpha \subset M$, contain only one element of the group G, *i. e.* G is Abelian.

The multigroup formed by the complete conjugate sets of elements of an Abelian group G is a group evidently coinciding with G.

For the sequel we need the following

Lemma. If Z is a multigroup formed by the complete conjugate sets of elements of a certain group G, then it is possible to establish between the normal divisors of the group G and the submultigroups of Z a one-to-one correspondence, under which to a normal divisor of G there corresponds that submultigroup of Z which is formed by the complete conjugate sets of G composing the normal divisor considered.

In fact, according to our definition of the submultigroup, the set of elements of G entering into the complete conjugate sets of elements forming the submultigroup of Z represents a normal divisor of the group G. Conversely, any normal divisor N of the group G represents an aggregate of certain complete conjugate sets of elements of G forming a certain submultigroup $\overline{Z}$ of the multigroup Z. In this case we say that the normal divisor N corresponds to the submultigroup $\overline{Z}$ and that $\overline{Z}$ corresponds to N.

Theorem 2. *If Z is a multigroup formed by the complete conjugate sets of elements of a group G, then the factor-multigroup $Z/\overline{Z}$ is isomorphic to the multigroup formed by the complete conjugate sets of elements of the factor-*

group G/N, where N is the normal divisor of G corresponding to the submultigroup Z.

In fact, let $\{C_\alpha\}$, where $\alpha \subset M$, be the complete conjugate sets of elements of the group G representing the elements of the multigroup Z, and let the normal divisor N correspond to the submultigroup $\bar{Z}$. By definition, the factor-multigroup $Z/\bar{Z}$ is the multigroup whose elements are the subsets of elements of Z of the form $\bar{Z}C_\alpha$, where $\alpha \subset M$, whereas the operation of multiplication is defined as the multiplication of subsets of Z indicated above. To prove the isomorphism it is obviously sufficient to correlate to the element $\bar{Z}C_\alpha$ of the factor-multigroup $Z/\bar{Z}$ the complete conjugate sets of elements of the factor-group G/N consisting of the elements of G/N, which are represented by cosets entering into $N\{C_\alpha\}$.

Let Z be a multigroup formed by the complete conjugate sets of elements of a certain group G, and $\bar{Z}$ — its submultigroup. From the lemma and the theorems 1 and 2 immediately follows that the factor-multigroup $Z/\bar{Z}$ is a group if and only if the normal divisor of G, which corresponds to $\bar{Z}$, includes the commutant of G.

Theorem 3. *The group G is solvable if and only if the multigroup formed by its complete conjugate sets of elements is an ultragroup.*

Proof. Let the multigroup Z formed by the complete conjugate sets of elements of the group G be an ultragroup, *i. e.* let it be possible to find a finite series of submultigroups

$$Z = Z_0 \supset Z_1 \supset \ldots \supset Z_m = e$$

such that the factor-multigroups Z_{i-1}/Z, $i = 1, 2, \ldots, m$, are groups and $Z_m = e$ is the unit of Z and, consequently, also the unit of G. Denote by $G = G_0, G_1, \ldots, G_m = e$ the normal divisors of the group G corresponding to the submultigroups $Z = Z_0, Z_1, \ldots, Z_m = e$. Evidently we have

$$G = G_0 \supset G_1 \supset \ldots \supset G_m = e$$

By Theorem 2 the multigroup formed by the complete conjugate sets of elements of the factor-group G_{i-1}/G_i is isomorphic to the group Z_{i-1}/Z, for $i = 1, 2, \ldots, m$; by Theorem 1 the factor-groups G_{i-1}/G_i, $i = 1, 2, \ldots, m$ are Abelian, and, consequently, the group G possesses a normal series with Abelian factors, *i. e.* G is solvable (2).

Conversely, if G is a solvable group, then its series of commutants, *i. e.*

$$G = \mathfrak{K}_0 \supset \mathfrak{K}_1 \supset \ldots \supset \mathfrak{K}_n = e,$$

in which every following subgroup is the commutant of the preceding one, terminates in the unit e of the group G.

Let Z be a multigroup formed by the complete conjugate sets of elements of the group G and $Z = Z_0, Z_1, \ldots, Z_n = e$, the submultigroups corresponding to the normal divisors $G = G_0, G_1, \ldots, G_n = e$ of the group G; evidently we have

$$Z = Z_0 \supset Z_1 \supset \ldots \supset Z_n = e$$

As the factor-groups $\mathfrak{K}_{i-1}/\mathfrak{K}_i$, $i = 1, 2, \ldots, n$, are Abelian, therefore by Theorems 1 and 2 the factor-multigroups Z_{i-1}/Z_i, $i = 1, 2, \ldots, n$, must be groups, *i. e.* Z is an ultragroup.

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MATHEMATICS

ON SEPARABLE *K*-SPACES

By A. G. PINSKER

(Communicated by A. N. Kolmogoroff, Member of the Academy, 9.X. 1944)

In the present note we consider the problem of representation of an arbitrary separable K-space (1) by means of spaces of measurable functions and of numerical sequences. In the case where a separable K-space is normed in such a way that certain conditions are satisfied (a space of the type KB_4, which in L. Kantorovitch's terminology is of the type B_2), I have already (2) established the following proposition:

Every separable space X of the type KB_4 is normally contained in L, or in l, or in their disjunctive sum $L+l$; in the first case X is continuous, in the second discrete, in the third it falls into the disjunctive sum of its subspaces: the continuous and the discrete.

Under L and l we understand, respectively, the space of summable functions defined in the interval $[0,1]$ and the space of numerical sequences $\{\xi^{(k)}\}$ such that $\sum_{k=1}^{\infty} |\xi^{(k)}| < +\infty$ with the usual definitions of the operations of addition and multiplication by a real number, of the norm and of the semi-orderedness.

By the symbols S and s we shall denote, respectively, the K-space of measurable functions defined in the interval $[0,1]$ and the K-space of all possible numerical sequences.

The results formulated below may be obtained by means of the theorem on the decomposition of a K-space into the disjunctive sum of elementary spaces, which was announced in my preceding note (3).

1°. Let X be a separable K-space, *i. e.* a space which contains a denumerable everywhere dense set; X may be represented in the form of a disjunctive sum of elementary spaces: $X = \sum_{\xi} x_{\xi}$.

Theorem 1. a) *An elementary subspace X_ξ of a separable K-space X is a separable space*; b) *an extension $\tilde{X}_\xi$ of an elementary separable space is an (elementary) separable space*; c) *the set of unit elements of an elementary separable space X_ξ is separable*; d) *in the extension of an elementary separable space is always contained a separable normal subspace $L(\tilde{X}_\xi)$ of the type KB_5 with an extension isomorphic to $\tilde{X}_\xi$.*

Theorem 2. a) *A continuous separable elementary space is normally contained in S*; b) *a discrete separable elementary space is normally contained in s*; c) *a mixed separable elementary space is normally contained in the disjunctive sum $S+s$.*

The theorem follows from the fact that $L(\tilde{X}_\xi)$ is isomorphic to L, or l, or $L+l$, and the extensions of these spaces are isomorphic, respectively, to S, s and $S+s$.

Theorem 3. *Every separable K-space X may be represented in the form of a disjunctive sum of spaces of measurable functions S_ξ and of numerical sequences s_ξ*

$$X=\sum_\xi S_\xi+\sum_\xi s_\xi;\quad S_\xi\subset S,\quad s_\xi\subset s$$

the power of the set of summands is less than the power of continuum.

A stronger result may be obtained by means of the following obvious corollary of the continuum hypothesis

$$2^{\aleph_1}>2^{\aleph_0}$$

Theorem 4. *Every separable K-space is normally contained in the disjunctive sum $S+s$.*

Remark. If a K-space X satisfies the following condition: every bounded set of pairwise disjunctive elements of X is not more than denumerable, then the theorem 4 may be established without the use of the continuum hypothesis.

2°. Certain propositions of the general theory of K-spaces are connected with the consideration of only denumerable sets and sequences (for instance, the properties of the limit, the question of completeness, *etc.*) and, therefore, it is natural to propose the question on the possibility of including a denumerable set of elements of a given space into its separable subspace. This question is comparatively easily solved in the case of a regular space.

Theorem 5. *Whatever be a denumerable set E_0 of a regular K-space X, there exists a regular subspace $X_0\subset X$ containing E_0.*

In the general case holds

Theorem 6. *Whatever be a denumerable set E_0 of an arbitrary K-space X, there exists a subspace $X_0\subset X$ falling into the disjunctive sum of separable elementary spaces.*

The importance of these last theorems consists in that the «local» properties of K-spaces may be investigated by means of the theory of functions of a real variable.

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FLUID DYNAMICS

CHARACTERISTICS OF A FULLY DEVELOPED TURBULENT MOTION FOR LARGE REYNOLDS' NUMBERS

By E. M. MINSKY

(Communicated by A. N. Kolmogoroff, Member of the Academy, 23. XII. 1944)

Of the heretofore published experiments on the kinematic characteristics of turbulent flows most were carried out at comparatively small Reynolds' numbers.

In order to add new data to the available information the present author undertook to measure the mean square value of the fluctuations of velocity $\sqrt{\overline{u^2}}$ in a long tube of rectangular cross-section 200×1200 mm, conveying an air current of great velocity. The walls of the tube were of mirror glass.

The measurements were made with a compensated hot-wire anemometer in a section 20 metres (100 heights of the tube) away from the entrance cone. This distance is sufficient for the creation of a fully developed turbulent flow.

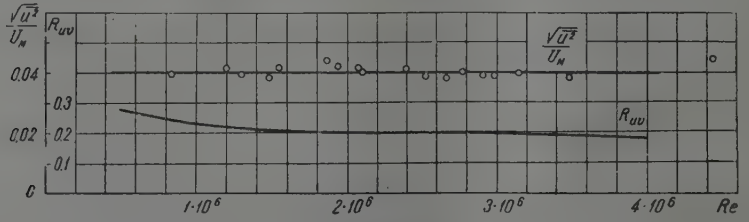

Fig. 1

The measurements were made at the axis of the tube with the average velocity of the flow ranging within 10 to 64 m/sec., which corresponds to the variation of Reynolds' numbers (referred to the velocity at the axis U_M and the half-height of the tube $H = 100$ mm) from $80 \cdot 10^3$ to $440 \cdot 10^3$. The results of the experiment, brought out by Fig. 1, show that the ratio of the mean square deviation to the maximum mean velocity $\sqrt{\overline{u^2}}/U_M$ is constant over the whole range of Reynolds' numbers $\mathrm{Re} = U_M H/\nu$.

Fig. 2 illustrates the results obtained by measuring the fluctuations of velocity in the cross-section of the tube for a mean velocity of about 50 m/sec., which corresponds to a Reynolds' number equal to $345 \cdot 10^3$. As may be seen from it, the intensity of turbulence varies along the cross-section of the flow from 0.04 (at the axis) to 0.07 (near the wall).

Alongside of turbulence measurements surface friction measurements were carried out in the same tube within a wide range of Reynolds' numbers. Assuming a linear law for the variation of friction along the vertical, let us compute a quan-

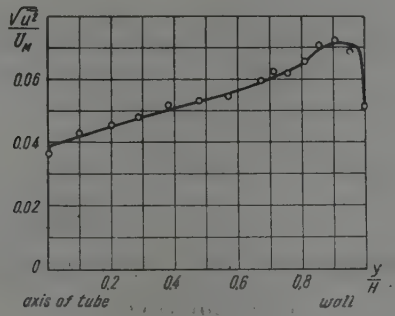

Fig. 2

tity analogous to the coefficient of correlation between the longitudinal and transverse velocity from the following formula

$$R_{uv}=\frac{\overline{uv}}{\overline{u^2}}=\frac{\tau}{\rho\overline{u^2}}=\frac{y}{H}\,\frac{C_{f_0}\frac{1}{2}\rho U_M^2}{\rho\overline{u^2}}=\frac{1}{2}\,C_{f_0}\frac{U_M^2}{\overline{u^2}}\,\frac{y}{H}$$

where u and v are the longitudinal and the transverse component of the fluctuations of velocity, respectively; U_M is the mean velocity at the tube axis; τ is turbulent friction; C_{f_0} is the coefficient of friction; ρ is the density of the fluid; y/H is the vertical coordinate referred to the half-height. A line above a letter denotes averaging with respect to time.

For the quantity C_{f_0} the following interpolation formula was obtained

$$C_{f_0}=\frac{0.0693}{(\lg \mathrm{Re})^2}$$

Assuming, in accordance with Fig. 1, that the intensity of turbulence $\sqrt{\overline{u^2}}/U_M$ does not depend on the value of Re, I have calculated the values of R_{uv} from the above formulae for $y/H=0.5$. The results of the calculations, represented in Fig. 1, show the variation of the coefficient R_{uv} as a function of Reynolds' number to coincide with the variation of the friction coefficient.

The turbulence measurements were carried out under participation of G. G. Zakharov, and in measuring friction N. N. Fomina lent a helping hand.

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THEORY OF ELASTICITY

REFLECTION AND REFRACTION OF NON-STATIONARY ELASTIC WAVES

By V. GOGOLADZE

(Communicated by N. N. Lusin, Member of the Academy, 31. X. 1944)

The problem of reflection and refraction of elastic waves has been the subject of many investigations in the domains of dynamic theory of elasticity and seismology. Their authors have been concerned with particular problems of different kind. In a fundamental work by Green (1), for instance, the elastic media are supposed to have equal Lamé constants, but different densities. Yet with this assumption made the question of Rayleigh waves propagating along the surface of separation between two elastic media is left out of consideration. In a complete manner the problem of refraction of elastic waves has only been solved for a half-space in the works by Lamb (2), Smirnov and Soboleff (3) and Naryshkina(4).

The present paper and the one to follow contain a full treatment of the elastic wave reflection and refraction problem for arbitrary elastic media that are in contact along a plane. Also a theory of Rayleigh boundary waves is developed in them.

Take the coordinate plane OXZ for the surface of separation between two media: region I ($Y>0$) with Lamé constants λ_1,μ_1 and density ρ_1, and region II with Lamé constants λ_2,μ_2 and density ρ_2.

Suppose the vibrations to be independent of the coordinate z and polarized in the plane OXY.

The problem of reflection and refraction of a linearly polarized transverse wave perpendicular to the plane OXY is solved in a rather elementary way (5).

The displacement of a particle is defined by the formulae (6)

$$u_j=\frac{\partial\varphi_j}{\partial x}+\frac{\partial\psi_j}{\partial y}\,;\qquad v_j=\frac{\partial\varphi_j}{\partial y}-\frac{\partial\psi_j}{\partial x} \tag{1}$$

where the potentials φ_j, ψ_j of the longitudinal and transverse waves satisfy the equations

$$\begin{gathered}\Delta\varphi_j=\frac{\partial^2\varphi_j}{\partial x^2}+\frac{\partial^2\varphi_j}{\partial y^2}=\frac{1}{a_j^2}\frac{\partial^2\varphi_j}{\partial t^2}\\ \Delta\psi_j=\frac{\partial^2\psi_j}{\partial x^2}+\frac{\partial^2\psi_j}{\partial y^2}=\frac{1}{b_j^2}\frac{\partial^2\psi_j}{\partial t^2}\\ j=1,(y>0);\ \ j=2,(y<0)\end{gathered} \tag{2}$$

The boundary conditions on the plane $y=0$ define the continuity of stresses and displacements

$$\begin{gathered}\left(\frac{a_1^2}{b_1^2}-2\right)\Delta\varphi_1+2\frac{\partial^2\varphi_1}{\partial y^2}-2\frac{\partial^2\psi_1}{\partial x\,\partial y}-\sigma\left[\left(\frac{a_2^2}{b^2}-2\right)\Delta\varphi_2+2\frac{\partial^2\varphi_2}{\partial y^2}-2\frac{\partial^2\varphi_2}{\partial x\,\partial y}\right]=0\\ 2\frac{\partial^2\varphi_1}{\partial x\,\partial y}+\frac{\partial^2\psi_1}{\partial y^2}-\frac{\partial^2\psi_1}{\partial x^2}-\sigma\left[2\frac{\partial^2\varphi_2}{\partial x\,\partial y}+\frac{\partial^2\psi_2}{\partial y^2}-\frac{\partial^2\psi_2}{\partial x^2}\right]=0\end{gathered} \tag{3}$$

$$\frac{\partial\varphi_1}{\partial x}+\frac{\partial\psi_1}{\partial y}-\left[\frac{\partial\varphi_2}{\partial x}+\frac{\partial\psi_2}{\partial y}\right]=0$$
$$\frac{\partial\varphi_1}{\partial y}-\frac{\partial\psi_1}{\partial x}-\left[\frac{\partial\varphi_2}{\partial y}-\frac{\partial\psi_2}{\partial x}\right]=0 \qquad (3)$$

where $\sigma=\mu_2/\mu_1$, the ratio of shearing moduli, has an arbitrary value.

The velocities of the longitudinal and transverse waves

$$a_j=\sqrt{\frac{\lambda_j+2\mu_j}{\rho_j}}>b_j=\sqrt{\frac{\mu_j}{\rho_j}}, \quad j=1, 2 \qquad (4)$$

take arbitrary values.

Let us consider the problem of reflection and refraction of a transverse wave with potential

$$\psi=\Psi\left(pt+\alpha x+y\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\right), \quad y>0 \qquad (5)$$

where $p^2/b_1^2-\alpha^2>0$, the velocity distribution being $b_1<b_2<a_2<a_1$. The other cases of velocity distribution are treated in an analogous manner.

The phase velocity c of the wave (5) along the OX-axis satisfies the inequality

$$|c|=\left|\frac{p}{\alpha}\right|>b_1 \qquad (6)$$

The real function $\psi(x)$ is continuous together with its derivative $\psi'(x)$ and fulfils the condition $|x^2\psi''(x)|<M$, where $M>0$ is a constant. This class of functions is known ([7]) to be representable in the form

$$\psi=\Psi\left(pt+\alpha x+y\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\right)=$$
$$=\frac{1}{2}\left[f_1\left(pt+\alpha x+y\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\right)+f_2\left(pt+\alpha x+y\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\right)\right] \qquad (7)$$

where $f_1(z)$ is regular in the upper complex half-plane and has in it a bounded derivative $f_1'(z)$. The function $f_2(z)$ is regular and its derivative $f_2'(z)$ is bounded in the lower half-plane, $f_1(z)=\overline{f_2(z)}$, and on the real axis we have $\operatorname{Re} f_1=\operatorname{Re} f_2=\psi$.

The problem of reflection and refraction will be solved for the functions f_1 and f_2 separately. Let

$$\psi=f_1\left(pt+\alpha x+y\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\right) \qquad (8)$$

We are going to consider four cases: $|c|>a_1$, $a_2<|c|<a_1$, $b_2<|c|<a_2$ and $b_1<|c|<b_2$.

1. In case $|c|>a_1$, the problem is solved in an elementary way by means of homogeneous (real) plane waves ([1]).

2. Suppose now that the phase velocity satisfies the inequality $a_2<|c|<a_1$. In this case the angle of reflection of the transverse wave is an imaginary quantity.

The potentials of reflected and refracted waves are sought in the form

$$\left.\begin{aligned}\varphi_1&=\Phi_1\left(pt+\alpha x+iy\sqrt{\alpha^2-\frac{p^2}{a_1^2}}\right)\\ \psi_1&=\Psi_1\left(pt+\alpha x-y\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\right)\end{aligned}\right\} y>0$$
$$\left.\begin{aligned}\varphi_2&=\Phi_2\left(pt+\alpha x+y\sqrt{\frac{p^2}{a_2^2}-\alpha^2}\right)\\ \psi_2&=\Psi_2\left(pt+\alpha x+y\sqrt{\frac{p^2}{b_2^2}-\alpha^2}\right)\end{aligned}\right\} y<0 \qquad (9)$$

Φ_1 is an unhomogeneous (complex) plane wave. For the potentials φ_j, ψ_j $(j=1, 2)$ we obtain, by virtue of the boundary conditions (3) and (8), the following values

$$\begin{aligned}
\Phi_1 &= E_1^* f_1\left(pt+\alpha x+iy\sqrt{\alpha^2-\frac{p^2}{a_1^2}}\right)\\
\Phi_2 &= E_3^* f_1\left(pt+\alpha x+y\sqrt{\frac{p^2}{a_2^2}-\alpha^2}\right)\\
\Psi_1 &= E_2^* f_1\left(pt+\alpha x-y\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\right)\\
\Psi_2 &= E_4^* f_1\left(pt+\alpha x+y\sqrt{\frac{p^2}{b_2^2}-\alpha^2}\right)
\end{aligned} \tag{10}$$

where the E_k^* $(k=1, 2, 3, 4)$ are obtained from the coefficients of intensity of reflected and refracted homogeneous waves E_k [8] by substituting the radical $\sqrt{p^2/a_1^2-\alpha^2}$ with $-i\sqrt{\alpha^2-p^2/a_1^2}$.

Similarly the solution of the problem for

$$\psi = f_2\left(pt+\alpha x+y\sqrt{p^2/b_1^2-\alpha^2}\right)$$

leads to the following result

$$\begin{aligned}
\Phi_1^{(1)} &= \overline{E}_1^* f_2\left(pt+\alpha x-iy\sqrt{\alpha^2-\frac{p^2}{a_1^2}}\right)\\
\Phi_2^{(1)} &= \overline{E}_3^* f_2\left(pt+\alpha x+y\sqrt{\frac{p^2}{a_2^2}-\alpha^2}\right)\\
\Psi_1^{(1)} &= \overline{E}_2^* f_2\left(pt+\alpha x-y\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\right)\\
\Psi_2^{(1)} &= \overline{E}_4^* f_2\left(pt+\alpha x+y\sqrt{\frac{p^2}{b_2^2}-\alpha^2}\right)
\end{aligned} \tag{11}$$

where $\overline{E_k^*}$ are obtained from E_k [8] by substituting $\sqrt{p^2/a_1^2-\alpha^2}$ with $i\sqrt{\alpha^2-p^2/a_2^2}$. In virtue of (7) and also because E_k^* and $\overline{E}_k^*$, $f_1(z)$ and $f_2(z)$ are conjugate complex numbers, the solution of the problem in the interval (a_2, a_1) may be written as follows

$$\left.\begin{aligned}
\varphi_1 &= \operatorname{Re}\left[f_1\left(pt+\alpha x+iy\sqrt{\alpha^2-\frac{p^2}{a_1^2}}\right)\right]\\
\psi_1 &= \operatorname{Re}\left[f_1\left(pt+\alpha x+y\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\right)+\right.\\
&\quad \left.+E_2^* f_1\left(pt+\alpha x-y\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\right)\right]
\end{aligned}\right\} \quad y>0 \tag{12}$$

$$\left.\begin{aligned}
\varphi_2 &= \operatorname{Re}\left[E_3^* f_1\left(pt+\alpha x+y\sqrt{\frac{p^2}{a_2^2}-\alpha^2}\right)\right]\\
\psi_2 &= \operatorname{Re}\left[E_4^* f_1\left(pt+\alpha x+y\sqrt{\frac{p^2}{b_2^2}-\alpha^2}\right)\right]
\end{aligned}\right\} \quad y<0$$

In the particular case of a periodic incident wave

$$\begin{aligned}
\psi &= A\cos\left[pt+\alpha x+y\sqrt{\frac{p^2}{b_1}-\alpha^2}\right] =\\
&= \frac{A}{2}\left[e^{i\left(pt+\alpha x+y\sqrt{p^2/b_1^2-\alpha^2}\right)}+e^{-i\left(pt+\alpha x+y\sqrt{p^2/b_1^2-\alpha^2}\right)}\right]
\end{aligned} \tag{13}$$

the potential of reflected longitudinal disturbance has the following form

$$\varphi_1 = e^{-y\sqrt{\alpha^2 - p^2/a_1^2}} \operatorname{Re} A E_1^* e^{i(pt+\alpha x)}, \quad y > 0 \tag{14}$$

It may be seen that here the damping of vibrations follows an exponential law.

3. If the phase velocity satisfies the inequality $b_2 < |c| < a_2$, we have

$$\left.\begin{aligned} \varphi_1 &= \operatorname{Re}\left[E_1^{(1)} f_1\left(pt+\alpha x+iy\sqrt{\alpha^2-\frac{p^2}{a_1^2}}\right)\right] \\ \psi_1 &= \operatorname{Re}\left[f_1\left(pt+\alpha x+y\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\right)+\right. \\ &\left.+E_2^{(1)} f_1\left(pt+\alpha x-y\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\right)\right] \end{aligned}\right\} \quad y>0 \tag{15}$$

$$\left.\begin{aligned} \varphi_2 &= \operatorname{Re}\left[E_3^{(1)} f_1\left(pt+\alpha x-iy\sqrt{\alpha^2-\frac{p^2}{a_2^2}}\right)\right] \\ \psi_2 &= \operatorname{Re}\left[E_4^{(1)} f_1\left(pt+\alpha x+y\sqrt{\frac{p^2}{b_2^2}-\alpha^2}\right)\right] \end{aligned}\right\} \quad y<0 \tag{16}$$

The values of $E_k^{(1)}$ are obtained from E_k^* by substituting

$$\sqrt{p^2/a_2^2-\alpha^2} \text{ with } -i\sqrt{\alpha^2-p^2/a_2^2}.$$

4. For $b_1 < |c| < b_2$ the solution of the problem is written as follows

$$\begin{aligned} \varphi_1 &= \operatorname{Re}\left[E_1^{(2)} f_1\left(pt+\alpha x+iy\sqrt{\alpha^2-\frac{p^2}{a_1^2}}\right)\right] \\ \psi_1 &= \operatorname{Re}\left[f_1\left(pt+\alpha x+y\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\right)+E_2^{(2)} f_1\left(pt+\alpha x-y\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\right)\right] \\ \varphi_2 &= \operatorname{Re}\left[E_3^{(2)} f_1\left(pt+\alpha x-iy\sqrt{\alpha^2-\frac{p^2}{a_2^2}}\right)\right] \\ \psi_2 &= \operatorname{Re}\left[E_4^{(2)} f_1\left(pt+\alpha x-iy\sqrt{\alpha^2-\frac{p^2}{b_2^2}}\right)\right] \end{aligned} \tag{17}$$

where the $E_k^{(2)}$ are obtained from $E_k^{(1)}$ by substituting $\sqrt{p^2/b_2^2-\alpha^2}$ with $-i\sqrt{\alpha^2-p^2/b_2^2}$. In the case of longitudinal wave the problem is investigated in a similar manner.

To obtain a complete solution of the reflection and refraction problem we have to investigate the nature of the obtained waves and the possibility of the existence of vibrations when $|c| < b_1$ and when the phase velocity c takes complex values. This problem is connected with that of the existence of Rayleigh boundary waves, which play a fundamental rôle in the propagation of seismic radiation in the interior of the Earth and at its surface.

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THEORY OF ELASTICITY

ON MAXWELL'S AND MORERA'S FORMULAE IN THE THEORY OF ELASTICITY

By R. O. KUZMIN

(Communicated by L. S. Leibenson, Member of the Academy, 28. I. 1945)

§ 1. In the case of equilibrium of an elastic body in the absence of volume forces the stress components satisfy the following equations due to Cauchy

$$\left.\begin{aligned} \frac{\partial X_x}{\partial x}+\frac{\partial X_y}{\partial y}+\frac{\partial X_z}{\partial z}&=0\\ \frac{\partial Y_x}{\partial x}+\frac{\partial Y_y}{\partial y}+\frac{\partial Y_z}{\partial z}&=0\\ \frac{\partial Z_x}{\partial x}+\frac{\partial Z_y}{\partial y}+\frac{\partial Z_z}{\partial z}&=0 \end{aligned}\right\} \qquad (1)$$

Maxwell expressed the six unknown components through three auxiliary functions in such a way that the Cauchy equations are satisfied for every choice of these functions. Maxwell's formulae are:

$$\left.\begin{aligned} X_x&=\frac{\partial^2\varphi_3}{\partial y^2}+\frac{\partial^2\varphi_2}{\partial z^2}, \quad Z_y=Y_z=-\frac{\partial^2\varphi_1}{\partial y\,\partial z}\\ Y_y&=\frac{\partial^2\varphi_1}{\partial z^2}+\frac{\partial^2\varphi_3}{\partial x^2}, \quad Z_x=X_z=-\frac{\partial^2\varphi_2}{\partial x\,\partial z}\\ Z_z&=\frac{\partial^2\varphi_2}{\partial x^2}+\frac{\partial^2\varphi_1}{\partial y^2}, \quad X_y=Y_x=-\frac{\partial^2\varphi_3}{\partial x\,\partial y} \end{aligned}\right\} \qquad (2)$$

Morera found another form of solutions of the equations (1). His formulae are

$$\left.\begin{aligned} X_x&=\frac{\partial^2\psi_1}{\partial y\,\partial z}, \quad Y_y=\frac{\partial^2\psi_2}{\partial x\,\partial z}, \quad Z_z=\frac{\partial^2\psi_3}{\partial x\,\partial y}\\ Y_z&=Z_y=-\frac{1}{2}\frac{\partial}{\partial x}\left(-\frac{\partial\psi_1}{\partial x}+\frac{\partial\psi_2}{\partial y}+\frac{\partial\psi_3}{\partial z}\right)\\ X_z&=Z_x=-\frac{1}{2}\frac{\partial}{\partial y}\left(\frac{\partial\psi_1}{\partial x}-\frac{\partial\psi_2}{\partial y}+\frac{\partial\psi_3}{\partial z}\right)\\ X_y&=Y_x=-\frac{1}{2}\frac{\partial}{\partial z}\left(\frac{\partial\psi_1}{\partial x}+\frac{\partial\psi_2}{\partial y}-\frac{\partial\psi_3}{\partial z}\right) \end{aligned}\right\} \qquad (3)$$

The direct substitution of (2) and (3) into the equations (1) shows that for every choice of the functions φ and ψ the formulae of Maxwell and Morera represent a solution of the equations of Cauchy.

In this connexion the question naturally arises as to the generality of the solutions so obtained. In the present note I prove that the solutions of

Maxwell and Morera possess a complete generality, if the stress components satisfy certain conditions of a very general nature.

§ 2. Supposing the stress components to be integrable functions of the coordinates, we may put

$$Y_z = -\frac{\partial^2 \psi_1}{\partial y\, \partial z}\,, \quad X_z = -\frac{\partial^2 \psi_2}{\partial z\, \partial x}\,, \quad X_y = -\frac{\partial^2 \psi_3}{\partial x\, \partial y} \tag{4}$$

Hence we may find the functions ψ_1, ψ_2 and ψ_3, whatever the integrable functions Y_z, X_z, X_y in the equations (1). For given Y_z, X_z, X_y the functions ψ_1, ψ_2 and ψ_3 may be determined in many ways. Substituting (4) in (1), we obtain

$$\left.\begin{aligned} \frac{\partial X_x}{\partial x} - \frac{\partial^3 \psi_3}{\partial x\, \partial y^2} - \frac{\partial^3 \psi_2}{\partial x\, \partial z^2} &= 0 \\ -\frac{\partial^3 \psi_3}{\partial x^2\, \partial y} + \frac{\partial Y_y}{\partial y} - \frac{\partial^3 \psi_1}{\partial y\, \partial z^2} &= 0 \\ -\frac{\partial^3 \psi_2}{\partial x^2\, \partial z} - \frac{\partial^3 \psi_1}{y^2\, \partial z} + \frac{\partial Z_z}{\partial z} &= 0 \end{aligned}\right\} \tag{5}$$

Integrating these equalities by x, y and z respectively, we find

$$\left.\begin{aligned} X_x &= \frac{\partial^2 \psi_3}{\partial y^2} + \frac{\partial^2 \psi_2}{\partial z^2} + \omega_1 (y, z) \\ Y_y &= \frac{\partial^2 \psi_3}{\partial x^2} + \frac{\partial^2 \psi_1}{\partial z^2} + \omega_2 (x, z) \\ Z_z &= \frac{\partial^2 \psi_2}{\partial x^2} + \frac{\partial^2 \psi_1}{\partial y^2} + \omega_3 (x, y) \end{aligned}\right\} \tag{6}$$

Herein ω_1, ω_2, ω_3 denote certain functions of the corresponding variables. If not only the stress components, but also their derivatives are integrable, then by an adequate choice of the functions ψ_1, ψ_2, ψ_3 in (4) we may achieve that ω_1, ω_2, ω_3 will be integrable by y, z and x respectively. Now we may put

$$\omega_1 = \frac{\partial^2 \theta_3 (y, z)}{\partial y^2}\,, \quad \omega_2 = \frac{\partial^2 \theta_1 (x, z)}{\partial z^2}\,, \quad \omega_3 = \frac{\partial^2 \theta_2 (x, y)}{\partial x^2} \tag{7}$$

We introduce another three functions by putting

$$\varphi_1 = \psi_1 + \theta_1, \quad \varphi_2 = \psi_2 + \theta_2, \quad \varphi_3 = \psi_3 + \theta_3 \tag{8}$$

Then we obtain the following equalities

$$\left.\begin{aligned} &\frac{\partial^2 \psi_1}{\partial y\, \partial z} = \frac{\partial^2 \varphi_1}{\partial y\, \partial z}\,, \quad \frac{\partial^2 \psi_2}{\partial x\, \partial z} = \frac{\partial^2 \varphi_2}{\partial x\, \partial z}\,, \quad \frac{\partial^2 \psi_3}{\partial x\, \partial y} = \frac{\partial^2 \varphi_3}{\partial x\, \partial y} \\ &\frac{\partial^2 \psi_2}{\partial z^2} = \frac{\partial^2 \varphi_2}{\partial z^2}\,, \quad \frac{\partial^2 \psi_3}{\partial x^2} = \frac{\partial^2 \varphi_3}{\partial x^2}\,, \quad \frac{\partial^2 \psi_1}{\partial y^2} = \frac{\partial^2 \varphi_1}{\partial y^2} \end{aligned}\right\} \tag{9}$$

Therefore, in the notations adopted, the formulae (4) and (6) assume the same form as the equalities (2). This shows that Maxwell's formulae yield a general solution of Cauchy's equations, which may represent every system of functions satisfying these equations and integrable together with their derivatives.

§ 3. Put, similarly, in the equations (1)

$$X_x = \frac{\partial^2 \varphi_1}{\partial y\, \partial z}\,, \quad Y_y = \frac{\partial^2 \varphi_2}{\partial x\, \partial z}\,, \quad Z_z = \frac{\partial^2 \varphi_3}{\partial x\, \partial y} \tag{10}$$

Without loss of generality we may write

$$X_y = \frac{\partial u}{\partial z}\,, \quad X_z = \frac{\partial v}{\partial y}\,, \quad Y_z = \frac{\partial w}{\partial x} \tag{11}$$

Then the equations (1) may be rewritten in the form

$$\left.\begin{aligned} \frac{\partial^2}{\partial y\,\partial z}\left(\frac{\partial \varphi_1}{\partial x}+u+v\right)&=0\\ \frac{\partial^2}{\partial x\,\partial z}\left(u+\frac{\partial \varphi_2}{\partial y}+w\right)&=0\\ \frac{\partial^2}{\partial x\,\partial y}\left(v+w+\frac{\partial \varphi_3}{\partial z}\right)&=0 \end{aligned}\right\} \tag{12}$$

Integrating these equations twice by the corresponding variables, we obtain the equalities

$$\left.\begin{aligned} \frac{\partial \varphi_1}{\partial x}+u+v&=a(y)+b(z)\\ v+\frac{\partial \varphi_2}{\partial y}+w&=c(x)+d(z)\\ v+w+\frac{\partial \varphi_3}{\partial z}&=e(x)+f(y) \end{aligned}\right\} \tag{13}$$

Solving them with respect to u, v, w, we find

$$\left.\begin{aligned} u+\frac{1}{2}\omega-\frac{\partial \varphi_1}{\partial x}&=\frac{1}{2}(-a-b+c+d+e+f)\\ v+\frac{1}{2}\omega-\frac{\partial \varphi_2}{\partial y}&=\frac{1}{2}(a+b-c-d+e+f)\\ w+\frac{1}{2}\omega-\frac{\partial \varphi_3}{\partial z}&=\frac{1}{2}(a+b+c+d-e-f) \end{aligned}\right\} \tag{14}$$

wherein

$$\omega=\frac{\partial \varphi_1}{\partial x}+\frac{\partial \varphi_2}{\partial y}+\frac{\partial \varphi_3}{\partial z}$$

Let us finally introduce another three functions ψ_1, ψ_2, ψ_3 determined by the equalities

$$\frac{\partial \psi_1}{\partial x}=\frac{\partial \varphi_1}{\partial x}-a-b;\quad \frac{\partial \psi_2}{\partial y}=\frac{\partial \varphi_2}{\partial y}-c-d;\quad \frac{\partial \psi_3}{\partial z}=\frac{\partial \varphi_3}{\partial z}-e-f \tag{15}$$

Hereby

$$\frac{\partial^2 \varphi_1}{\partial y\,\partial z}=\frac{\partial^2 \psi_1}{\partial y\,\partial z};\quad \frac{\partial^2 \varphi_2}{\partial z\,\partial x}=\frac{\partial^2 \psi_2}{\partial z\,\partial x};\quad \frac{\partial^2 \varphi_3}{\partial x\,\partial y}=\frac{\partial^2 \psi_3}{\partial x\,\partial y} \tag{16}$$

In virtue of (14), (15) and (16) the formulae (10) and (34) assume the form

$$\begin{aligned} X_x&=\frac{\partial^2 \psi_1}{\partial y\,\partial z},\quad Y_y=\frac{\partial^2 \psi_2}{\partial z\,\partial x},\quad Z_z=\frac{\partial^2 \psi_3}{\partial x\,\partial y}\\ Y_z&=X_y=-\frac{1}{2}\frac{\partial}{\partial x}\left(-\frac{\partial \psi_1}{\partial x}+\frac{\partial \psi_2}{\partial y}+\frac{\partial \psi_3}{\partial z}\right)\\ X_z&=Z_x=-\frac{1}{2}\frac{\partial}{\partial y}\left(\frac{\partial \psi_1}{\partial x}-\frac{\partial \psi_2}{\partial y}+\frac{\partial \psi_3}{\partial z}\right)\\ X_y&=Y_x=-\frac{1}{2}\frac{\partial}{\partial z}\left(\frac{\partial \psi_1}{\partial x}+\frac{\partial \psi_2}{\partial y}-\frac{\partial \psi_3}{\partial z}\right) \end{aligned}$$

This shows that Morera's solution possesses the required generality as well, *i. e.* that every solution of Cauchy's equations may be represented by Morera's formulae, if only the stress components satisfy certain conditions of integrability which follow from the proof given above and whose formulation we omit for want of space.

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ASTRONOMY

HEIGHTS AND VELOCITIES OF METEORS

By B. J. LEVIN

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1. A few years ago the long mooted problem of meteor velocities seemed to have been solved at last owing to observations made with the vibrating mirror. It appeared from these observations that an overwhelming majority of the sporadic meteors moved with hyperbolic velocities and hence arrived in the solar system from the interstellar space (1). But it soon turned out that observations with the aid of a vibrating mirror are fraught with considerable systematic errors (2), and the methods of their treatment are unreliable, leading to conflicting results. At the same time Wylie (3) and Watson(4) proved the unreality of hyperbolic velocities for fireballs and bright meteors from the Nissel-Hoffmeister catalogue. Treatment of duplicate meteor photographs taken at the Harvard Observatory by means of cameras provided with rotating shutters showed that all of the meteors photographed had elliptic velocities (5). Now it is just the bright meteors that appear on the photographs and the prevailing opinion regarding them was that they moved with hyperbolic velocities. So American astronomers came to th conclusion that by far most of the meteors if not all belonged to the solar system.

Quite recently McIntosh (6) and somewhat earlier Öpik (7) have proved that velocity estimates based on visual observations are accurate enough. Therefore, and also because the visual observations embrace at once the bright and the ordinary meteors, they may, no doubt, be turned to good account in attempting to solve the problem of meteor velocities.

2. In 1942 Porter published a catalogue (8) of heights and velocities for 289 meteors, consisting mainly of old observations treated by a novel, more perfect method. The same method was employed in treating the observations included in the current catalogues of the British Astronomical Association. Three of these catalogues (9) including 108 observations were added to Porter's catalogue as a material for statistical treatment, the results of which are referred to below. Ten meteors were discarded because it was certain that their heights and velocities had been estimated erroneously, and another three because their radiant had been slightly under the horizon. So the treated material contained 384 meteors.

For each meteor in the catalogue we have its theoretical geocentric velocity V which corresponds to the parabolic heliocentric velocity. By comparing V with the observed velocity S one can determine the character of the orbit of a meteor body. For the 324 meteors for which the estimates of S are available we get

$$\overline{V} = 50 \text{ km/sec.}, \quad \overline{S} = 40 \text{ km/sec.}$$

On the average the observed velocity is smaller than that corresponding to the parabolic velocity for the same meteor. And trying different characteristics as a basis for grouping meteors, we still find $\overline{V} > \overline{S}$ in a vast majority of cases.

Photographs taken with a rotating shutter show, in accordance with the physical theory of meteors, that retardation mainly occurs at the end of the visible path of the meteor in the atmosphere. So the average velocity on the visible portion of the path—the observed velocity—must be very near to the velocity outside the atmosphere.

Therefore the results referred to may be interpreted to mean that meteoric bodies move along elliptical orbits and are members of the solar system. There is no ground to believe in the reality of the cases of hyperbolic velocity, which make up 24 per cent in the total of records. They may well be accounted for by the unavoidable errors of observation.

The heliocentric velocity of sporadic meteors with retrograde motion is about 25—30 km/sec. on the average, while the meteors overtaking the Earth have nearly parabolic velocities. If the latter move slowly, they probably produce no meteoric phenomenon.

3. Table 1 gives the distribution of meteors according to elongation ε of the radiant from the apex of the orbital motion of the Earth. Here N is the distribution of all meteors. N' is the distribution of sporadic meteors (excluded are the meteors belonging to the Perseids, Leonids, Eta Aquarids and Orionids). Taking into account the areas F of the circular zones of the celestial sphere corresponding to different intervals of ε (and taking no account of the slight influence of the difference in the duration of the stay over the horizon, or of the variations of the altitude above the horizon for the different portions of these zones), we obtain the density of radiants N'/F at different distances from the apex.

Table 1

$\varepsilon°$	N	N'	F	N'/F
0— 9	4	0	0.014	0
10— 19	27	7	0.043	163
20— 29	39	14	0.073	192
30— 39	54	14	0.098	143
40— 49	60	15	0.123	122
50— 59	26	22	0.141	156
60— 79	63	63	0.326	193
80— 99	61	61	0.347	176
100—119	28	28	0.327	86
120—139	18	18	0.268	67
140—159	2	2	0.177	11
160—180	2	2	0.063	32

From the anti-apex to apex the density increases as much as nearly ten times. Theoretically, if sporadic meteors move with equal abundance in all directions with parabolic velocity, this increase should be as great as 1000—3000 times. Hence the number of orbits with direct motion is hundrends of times as large as the number of orbits with retrograde motion.

In the immediate vicinity of the apex the radiant density falls off sharply down to zero. This is probably because the meteoric bodies that happened to have their aphelion (or perihelion) lying exactly on the Earth orbit were soon captured by the Earth, and are practically exhausted at present.

The minimum density of the radiant about $\varepsilon \approx 45°$ may be accounted for by the scarcity of the orbits crossing the Earth orbit at right angles (this is possible only for orbits with a large inclination to the ecliptic). From the position of the minimum the average heliocentric velocity of the meteors appears to be about 30 km/sec., which agrees well with the results of the direct estimations referred to above.

4. The average characteristics given in Table 2 were obtained by grouping the meteors according to their height of disappearance (H_s) and brightness (M).

Table 2

		n	$\overline{H}_1$, km	$\overline{H}_2$, km	$\overline{V}$, km/sec.	$\overline{S}$, km/sec.
$H_2 < 60$ km	$M \geqslant 0^m$	15	82	53	38	25
	$M < 0^m$	26	90	48	35	33
$H_2 > 60$ km	$M \geqslant 0^m$	210	107	90	49	38
	$M < 0^m$	133	117	88	55	47

It may be seen from Table 2 that it is the more slowly moving meteoric bodies that penetrate deep into the atmosphere. On account of small velocity they also have low heights of appearance. The brightness of meteors is connected with their velocity and mass. Among the bright meteors and fireballs some have small and some have great heights of disappearance. But all move faster than do ordinary (with respect to brightness) meteors, and penetrate deeper into the atmosphere. This latter circumstance is evidence of their greater mass.

5. Table 3 gives the height distribution of meteors with not very gently slopping trajectories (the height or the radiant $a \geqslant 20°$). The distribution of the disappearance heights reveals not the slightest sign of the double maximum which is so sharply expressed in Lindeman and Dobson's curves built on the data of Denning's catalogues. A more careful analysis of Denning's catalogues ([10]) shows that the lower maximum is mainly due to the bright meteors. This is probably because of selection in the observation objects, owing to which the proportion of slow bright meteors with low heights of disappearance was greatly overestimated.

Table 3

H, km	n_{H_1}	n_{H_2}
30—39	0	4
40—49	0	10
50—59	1	19
60—69	7	20
70—79	16	52
80—89	22	79
90—99	51	73
100—109	70	55
110—119	74	7
120—129	42	7
130—139	21	1
140—149	13	2
150—159	5	0
160—169	3	0
170—179	4	0

6. Table 4 brings out the increase of height with increasing velocity for sporadic meteors (the height of the radiant $a \geqslant 20°$).

Table 4

$\overline{V}$, km/sec.	n	$\overline{H}_1$, km	$\overline{H}_2$, km	$\overline{M}$
17—19	7	104	70	— 1.7
20—29	46	98	74	0.0
30—39	50	100	76	+ 0.3
40—49	41	107	83	+ 0.3
50—59	25	104	84	+ 0.4
60—69	26	114	94	+ 0.6
70—79	6	128	100	— 0.5

In this table the speed of the meteors is characterized by the «theoretical» velocity V. An analogous table built on the observed velocity S as argument is by far less reliable. It appears that the true velocity of meteors is better characterized by the theoretical than by observed velocity.

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Remark added in proof-reading. In June 1945 a number of Monthly Notices of RAS containing a second paper by Porter was received in Moscow. His results in treating British meteor data will be compared with my results in a paper to be published in the Russian Astronomical Journal.

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PHYSICS

THE INFLUENCE OF THE CHEMICAL BOND ON THE SYMMETRY OF THE $K\alpha_{1,2}$-LINES IN THE X-RAY SPECTRUM OF THE ELEMENTS OF THE IRON GROUP

By E. E. WEINSTEIN and I. B. BOROVSKY

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The $K\alpha_{1,2}$-lines of the X-ray spectrum are due to $2p$—$1s$-transitions in the atom and, theoretically speaking, should be strictly symmetrical. At the same time experimental study of this group of lines for various elements ([1-4]) has shown that non-symmetrical lines with an index of asymmetry that increases from Sc to Fe and then decreases to Zn and Ga are obtained for atoms with $20<Z<30$, in which completion of the incomplete $3d$-levels is taking place. Although these observations have been accumulating in the literature for many years now, they have never attracted any special attention, and it was not until recently that they were subjected to systematic analysis by Snyder([5]) and one of us ([6]) for the case of the elements of the iron group.

A semi-quantitative treatment of the observed relationship that takes into account the exchange interaction between the $2p$- and $3d$-electrons, resulting in the splitting of the $2p$-levels and the appearance of an unsplit doublet, pointed to the obvious connexion of the phenomenon with the peculiarities in the structure of the $3d$-shell of the atoms and, in particular, made it possible to establish the dependence of the index of asymmetry of the lines upon the number of unpaired electrons in the $3d$-shell. In this connexion it was of great interest to study the influence of the chemical bond on the symmetry of the $K\alpha_{1,2}$-lines of the elements of the Fe-group in the simplest, most stable and definite compounds.

Heretofore all work on studying the shape of the lines was done on a double crystal spectrograph. In our work we studied the relative changes in the index of asymmetry of the $K\alpha_{1,2}$-lines of the elements in various chemical compounds with a bent crystal spectrograph as used by Cauchoix. The analyser in the spectrograph was a quartz plate 0.15 mm thick, cut perpendicular to the optical axis of the crystal and having a curvature of 880 mm in radius. The spectrum was photographed in the second order reflections from the $(10\bar{1}0)$ plane having a lattice constant $d = 4.24574$ Å on a flat plate inclined at an angle $\vartheta/2$ (where ϑ is the Bragg angle) with respect to the incident ray. Herewith the dispersion reached the value 7—9 XU/mm. The lines were photographed exactly in the centre of the plate. To reduce a minimum heating of the substance on the anticathode, the regime of the tube was 3—4 mA and 40 kV when the total spot was 100 mm wide. The time of exposure for the different substances was so chosen that the ratio of the $K\alpha_1$ to the $K\alpha_2$-lines on the photomicrograph was close to the theoretical value of 1 : 2 and varied from half an hour to two hours.

The geometrical conditions of focusing in the Cauchoix spectrograph gave rise

to certain discrepancies between our data and the results of measurements on a double crystal spectrograph.

The lines observed in our experiments were narrower (in eV) and more symmetrical than those cited in the literature*; what is more, the observed discrepancy, in full accord with the conclusions of the geometrical theory of the apparatus, diminished regularly with a decrease in θ (from Fe to Zn), ceasing to play any essential rôle in the region of angles corresponding to the $K\alpha$-lines of Cu and Zn. In these cases our values for the index of asymmetry were found to be practically the same as those cited in the literature, whereas for Fe and Co they were 25 and 10 per cent lower, respectively.

For the sake of clearness in conclusions we selected for investigation only the simplest, thermally most stable compounds, the valency of the atoms of the transition elements in which had been sufficiently well elucidated by chemical investigations. In most cases these were minerals of the simplest oxides of the elements of the iron group and Zn (hematite, ilmenite, willemite), or compounds specially prepared in the laboratory (NiO, $NiSO_4$). The spectra of the metals were obtained from corresponding powders that had been reduced in an oven with hydrogen.

The spectra obtained were photomicrographed with a Sviegbau microphotometer employing a magnification of 25. The measurements of the index of asymmetry of the lines were made with an Abbé comparator.

The values of the index of asymmetry cited were obtained by averaging the results of measurements of 12 individual curves. The mean error does not exceed 5—7 per cent of the measured value of the index of asymmetry.

The experimental results are given in Table 1.

Table 1

Element	Substance	Line	Index	Width in XU	Width in eV	Element	Substance	Line	Index	Width in XU	Width in eV
Fe	Fe	$K\alpha_1$	1.40	0.695	2.3	Ni	Ni	$K\alpha_1$	1.21	0.585	2.6
		$K\alpha_2$	0.97	0.700	2.3			$K\alpha_2$	1.21	0.523	2.4
	Fe_2O_3	$K\alpha_1$	1.30	0.840	2.8		$NiSO_4$	$K\alpha_1$	1.20	0.564	2.5
		$K\alpha_2$	0.87	0.790	2.6			$K\alpha_2$	1.21	0.606	2.7
	Fe_3O_4	$K\alpha_1$	1.39	0.760	2.5		NiO	$K\alpha_1$	1.20	0.590	2.6
		$K\alpha_2$	0.96	0.655	2.2			$K\alpha_2$	1.18	0.600	2.7
	$FeTiO_3$	$K\alpha_1$	1.40	0.810	2.7		Ni_2O_3	$K\alpha_1$	1.05	0.540	2.4
		$K\alpha_2$	0.99	0.760	2.5			$K\alpha_2$	1.07	0.583	2.6
Co	Co	$K\alpha_1$	1.30	0.675	2.6	Zn	Zn	$K\alpha_1$	1.07	0.670	4.0
		$K\alpha_2$	1.10	0.630	2.4			$K\alpha_2$	1.08	0.520	3.1
	$CoO + Co_2O_3$	$K\alpha_1$	1.40	0.720	2.8		ZnO	$K\alpha_1$	1.06	0.590	3.5
		$K\alpha_2$	1.16	0.635	2.7			$K\alpha_2$	1.00	0.560	3.3
							$ZnSiO_4$	$K\alpha$	1.04	0.502	3.0
								$K\alpha_2$	1.06	0.521	3.1
						Cu	Cu	$K\alpha_1$	1,07	0.601	3.1
								$K\alpha_2$	1.00	0.608	3.2

An analysis of the results makes it possible to draw the following conclusions:

1. A spectrograph with a bent crystal may be used to make comparative measurements of the index of asymmetry. The good reproducibility of the results ensures sufficiently high accuracy of the measurements.

* In the Cauchoix method the lines are sharply limited on the long-wave side, and their width on the short-wave side increases with an increase in the Bragg angle.

2. In the series of elements from Fe (26) to Zn (30), a systematic decrease in the index of asymmetry of the $K\alpha$-lines with a decrease in the atomic number of the element is observed. In the case of elements whose atoms have completed $3d$-levels, the $K\alpha$-lines are practically symmetrical.

3. In the case of elements with completed $3d$-levels (in our case Zn), the symmetry of the $K\alpha$-lines is not upset when the atoms enter into chemical compounds. Within the limits of experimental error the width of the line also remains constant therewith.

4. In the case of elements with defects in the $3d$-shell of the atom, the index of asymmetry of the lines is found to depend upon the valence of the atom in the compound. Herewith a change in valence of two units practically does not affect the symmetry of the $K\alpha$-lines. At the same time transition of the atom of the metal to the trivalent state (Fe_2O_3, Ni_2O_3) leads in the case of Fe and Ni to a decrease in the index of asymmetry, and in the case of Co to a slight increase in the index of asymmetry, owing to the presence in the investigated mixture of Co-oxides of a slight amount of Co_2O_3. The different direction of the change in the index of asymmetry of the $K\alpha_{1,2}$-lines upon transition to the trivalent state in comparison with the metal for Fe, Ni and Co is apparently connected with the different even number of electrons filling the $3d$-levels in the Fe, Ni and Co atoms.

5. In compounds or mixtures of compounds in which the atoms of the investigated element are present in different valence states, the symmetry of the observed X-ray $K\alpha$-lines is determined by the value of the index of asymmetry of the most asymmetrical line (Fe_3O_4, $FeTiO_3$ and $CoO+Co_2O_3$).

The experimentally observed regularities are in qualitatively good agreement with the earlier conceptions (6) to the effect that the value of the index of asymmetry of the $K\alpha$-lines in the spectrum of the elements of the Fe-group is connected with the characteristic interactions of the $3d$- and $2p$-electrons in the atoms of these elements, and essentially depends upon the number of unpaired electrons in the $3d$-shell. In line with this we resolved the observed $K\alpha$-lines into symmetrical components, whose number, as was to be expected, does not exceed two within the limits of experimental accuracy (5—7 per cent). For the $K\alpha_2$-lines of Co, whose index of asymmetry is only 10 per cent greater than unity, the accuracy of resolution is less, and the data cited for it are less reliable. Lines were resolved whose index of asymmetry corresponded closest to the mean value of the index of asymmetry of the lines for the given group of substances and the ratio of the intensities of $K\alpha_1$ to $K\alpha_2$ for which was found to be closest to the theoretical value of 1 : 2. Averaged results of measurements of such curves are given for the $K\alpha_1$- and $K\alpha_2$-lines in Tables 2 and 3.

In corresponding columns of the tables are given $\Delta\nu$, the distance in XU and eV between the maxima of the two lines into which the microphotometric curves, preliminarily magnified sixfold, were resolved; I_1/I_2, the ratio of the intensities of the two lines, measured from the maximum ordinate; and ΔW, the half-width of both components in XU and eV. The shape of the most intense symmetrical component was found to be satisfactorily described by the Hoyt formula $y = I_{max}/1+(x/b)^2$ in which I_{max} is the maximum ordinate, b is the half-width of a line at the height $I_{max}/2$, and x is the coordinate.

Striking is the relatively small difference in the eV values of $\Delta\nu$ for the three ferromagnetic elements and their practical identity for the metals and oxides of these elements*. Still more interesting is the fact that the ratio of the intensities I_1/I_2 is almost perfectly constant for the entire group of $K\alpha_{1,2}$-lines in the substances investigated. This, it seems to us, is an indication of the very close relationship of the factors giving rise to the asymmetry of the $K\alpha$-lines in elements with incomplete $3d$-levels.

* Upon passing to «absolute» values of the index of asymmetry (measured on a double crystal spectrograph), this should correspond to a decrease in $\Delta\nu$ upon transition from Fe to Ni, which for the $K\alpha$-lines, for example, may amount to 3.80, 3.70 and 3.00 eV for these elements, and 3.85, 3.60 and 3.00 eV, respectively, for the oxides, since, as our measurements for Ni showed, the changes in the index of asymmetry are practically proportional to the change in $\Delta\nu$.

Table 2

Substance and line	Mean index	$\Delta\nu\ K\alpha_1$ in XU	$\Delta\nu\ K\alpha_1$ in eV	I_1 / I_2	Mean width of the first line (ΔW_1) in XU	Mean width of the first line (ΔW_1) in eV	Mean width of the second line (ΔW_2) in XU	Mean width of the second line (ΔW_2) in eV
Ni $K\alpha_1$	1.23	0.57±0.07	2.56±0.3	0.15±0.03	0.47±0.1	2.1	0.31±0.1	1.4
Co $K\alpha_1$	1.32	0.89±0.07	3.44±0.3	0.16±0.03	0.58±0.1	2.2	0.52±0.1	2.0
Fe $K\alpha_1$	1.39	0.91±0.07	3.00±0.3	0.17±0.04	0.68±0.1	2.3	0.45±0.1	1.5
NiO $K\alpha_1$	1.22	0.60±0.07	2.70±0.3	0.22±0.04	0.47±0.1	2.1	0.33±0.1	1.5
CoO * $K\alpha_1$	1.41	0.85±0.07	3.28±0.3	0.20±0.03	0.59±0.1	2.3	0.47±0.1	1.8
Fe_2O_3 $K\alpha_1$	1.38	1.02±0.07	3.38±0.3	0.20±0.03	0.67±0.1	2.2	0.57±0.1	1.9
$FeTiO_3$ $K\alpha_1$	1.38	0.98±0.07	3.25±0.3	0.19±0.03	0.60±0.1	2.0	0.54±0.1	1.8

* Arbitrary, mixture of oxides.

Table 3

Substance and line	Index of asymmetry	$\Delta\nu\ K\alpha_1$ in XU	$\Delta\nu\ K\alpha_1$ in eV	I_1 / I_2
Fe $K\alpha_2$	0.97	—	—	—
Co $K\alpha_2$	1.10	0.67	2.6	0.11±0.02
Ni $K\alpha_2$	1.21	0.71	3.2	0.15±0.02

The appearance of $3d$—$2p$-interaction in the atoms of these elements leads to the splitting of the $2p$-levels and the appearance of a new doublet. The relative intensity of the lines of the latter is determined by the probability of the transition of the $2p$-electrons from these new levels to the $1s$-levels of the atoms.

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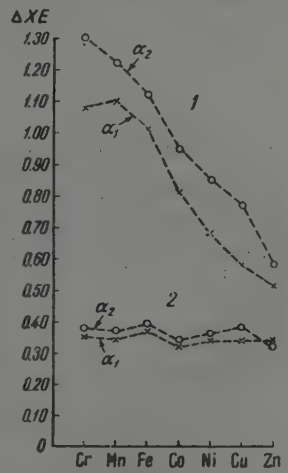
PHYSICS

HYPERFINE STRUCTURE OF SECONDARY X-RAY SPECTRA

By A. I. KRASNIKOV

(Communicated by A. I. Alikhanov, Member of the Academy, 19. XII. 1944)

The properties of the spin-doublet line $K\alpha_1\alpha_2$ have been studied in numerous experiments by different authors(1). In Compton and Allison' book we find the fullest data on the width of these lines (2). The figure compares the results of width measurements made by Allison and Parratt (2) with double crystal spectrometer (cathodic excitation) and by the author with single crystal spectrograph (the fluorescence method). It may easily be seen that the half-width of the lines in secondary X-ray spectra is far less in absolute value and scarcely varies with the atomic number of the element. From the fact that the half-width obtained for $K\alpha_1\alpha_2$-lines by Allison and Parratt is several times the natural half-width, Compton and Allison(2) have arrived at the conclusion that only the order of magnitude of the latter is estimated correctly by the theory.



1, Allison-Parratt;
2, Krasnikov

In 1942 while measuring the width of $K\alpha_1\alpha_2$-lines for Mn, Fe, Co, Ni, Cu and Zn, the author found that in some cases these lines have a peaked contour instead of running smoothly. This I interpreted to mean that the $\alpha_1\alpha_2$-lines of the spin-doublet were no single lines but at least double.

The assumption was corroborated in 1944 when I was able to obtain $K\alpha_1\alpha_2$ Cr_2O_3 doublets in which the α_1-line was resolved into two components—one, α_1', of short wave-length; the other, α_1'', of long.

In the adjoined table are given the results of accurate linear symmetrical measurements (3), by means of a comparator, of two doublets photographed one after another on a single film (double spectrograph of 25. V. 1944). Secondary anticathode, Cr_2O_3. All quantities are expressed here in millimetres, that is, the figures are those obtained directly with the comparator. The probable error of measurements is ± 0.002 mm, or ± 0.003 XU. Under conditions of the experiment one millimetre on the film is equivalent to 1.58 XU or to 3.7 volt. The width of the lines was measured from edge to edge, so the half-width is approximately two times smaller.

If the average for the width of the lines α_1' and α_1'' in the two photographs be taken as the most probable value, their half-width will be 0.095 XU. From additional, control, spectrograms obtained with a higher resolving

power the half-width of the lines α_1' and α_1'' is 0.10 XU for Cr_2O_3, 0.16 XU for Fe_3O_4, and 0.12 for the components α_2' and α_2'' Cr_2O_3. These approximate computations show that the average half-width of the lines α_1', α_2'', α_2' and α_2'' is very near to the half-width calculated on the basis of classical concepts ($\Delta\lambda' = 0.12$ XU $=$ const).

Nos. of spectrograms	Line width in mm		Doublet distance $\alpha_1-\alpha_2$ in mm	Width of the components in mm		Doublet distance $\alpha_1'-\alpha_1''$ in mm	Visible separation between lines $\alpha_1'-\alpha_1''$ in mm
	α_1	α_2		α_1'	α_1''		
5	0.668	0.593	2.498	0.281	0.266	0.311	0.113
6	0.654	0.599	2.493	0.276	0.261	0.299	0.110

The intensity ratio between the lines α_1' and α_1'', as revealed by photomicrometry with an accuracy of $\pm$ 10 per cent, is 1 to 1, and not 2 to 1 as in the case of a spin doublet. The doublet distance is eight times as small in $K\alpha_1'-\alpha_1''$ as in $K\alpha_1-\alpha_2$, whereas the width of the $\alpha_1'\alpha_1''$ lines is only 2—2.5 times smaller. The contour of the lines α_1', α_1'', α_2', and α_2'' has not yet been studied. In the $K\alpha_1'-\alpha_1''$ and $K\alpha_2'$ α_2'' doublets the variation of the doublet distance and line width is controlled by the same factors as in the spin doublet $K\alpha_1-\alpha_2$ (4).

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PHYSICS

CHARACTERISTIC ELECTRIC OSCILLATIONS OF A LOW PRESSURE MERCURY ARC

By B. L. GRANOVSKY and L. N. BYKHOVSKAYA

(Communicated by N. D. Papalexi, Member of the Academy, 9. I. 1945)

§ 1. The possibility of undamped electric oscillations arising in a circuit having neither capacity nor inductance is today a firmly established fact. A circuit consisting only of a source of a constant electromotive force, ohmic resistance and discharge gap, may generate stable oscillations of both comparatively low frequency (10^3—10^6 cycles) and ultra-high frequency (10^9—10^{10} cycles). The existence of oscillations of medium and high frequency during discharge in various gases has been established and the frequencies measured in a number of works ([1–8]). However, there is no data in the literature on the character of these oscillations, on their amplitude and on the mechanism of their origination.

We have made a systematic study of the characteristic natural electric oscillations of a mercury arc at vapour pressures from 0.2 to 50 μ Hg. The experiments were carried out in a vertical cylindrical tube containing only a liquid mercury cathode with a molybdenum ring for anchoring the cathode spot, and a flat molybdenum anode; the arc ignition needle was enclosed in a special side-tube. The pressure of the mercury vapour was regulated by a water bath; gaseous impurities were constantly pumped off. The tube was fed by a battery of accumulators having 120—300 V; the circuit did not contain any capacities or inductances besides the distribution parameters of the lead-in wires. The investigation of the oscillations was carried out with an electron oscillograph and an electronic voltmeter.

§ 2. Setting aside the oscillations of ultra-high frequencies, we were able to establish the existence of four different types of oscillations: 1) irregular, chaotic deviations of the voltage U from its normal value in the case of a freely moving cathode spot; 2) chaotic oscillations of higher frequencies in the case of an anchored cathode spot; 3) regular, *i. e.* strictly periodic oscillations in the range of low radio-frequencies (10^4—10^5 cycles per sec.); and 4) regular oscillations in the range of sound frequencies (3000 to 300 cycles and lower).

The results of observations we made of these different types of oscillations are now analysed in brief.

1. Chaotic Oscillations in the Case of a Freely Moving Cathode Spot. The character of the oscillations is chaotic, the height of the peaks and the length of the gaps between them fluctuate noticeably. The frequencies lie in the range 10^2—10^4 cycles. The oscillations are non-symmetrical: predominant are deviations of the voltage above the fundamental value ($\Delta U > 0$), and deviations of the current strength below the fundamental value

($\Delta i < 0$). The mean sweep of the voltage oscillations*(denoted by ΔU) may be as much as 15 V, and in the case of individual peaks, even higher. With an increase in i_{mean}, ΔU diminishes.

The dependence of ΔU upon the length of the discharge l, measured in a tube with a mobile cathode, is complex: between $l=2$ cm and $l=20$ cm ΔU twice passes through a maximum, never vanishing completely, no matter what the length is. The temperature of the walls (*i. e.* the pressure of the saturated vapour) does not have any noticeable effect. When the cathode spot is anchored, these oscillations cease.

Oscillations of this type may have the following mechanism. After each shift of the cathode spot, the current at first must pass through a layer of weakly ionized gas and hence the current falls. Since an ohmic resistance is included in the circuit, the voltage applied to the tube increases. As the gas gradually ionizes, the current mounts, while the voltage falls, until a new shift in the cathode spot occurs.

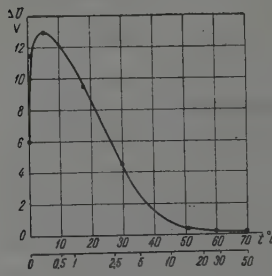

Fig. 1

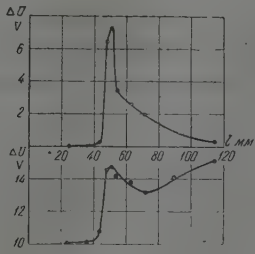

Fig. 2

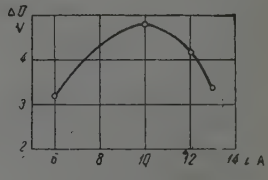

Fig. 3

This explains both the unilateral character of the oscillations ($\Delta U > 0$, $\Delta i < 0$) and their low frequency. Their independence of the temperature of condensation of mercury follows from the fact that the main rôle is played by the region in the vicinity of the cathode spot, where the vapour pressure is determined in the main by the flow of vapour from the latter and not by the temperature of the walls.

2. Chaotic Oscillations in the Case of an Anchored Spot. The frequency is considerably higher, in the range 10^4—10^5 cycles. The amplitude of the oscillations is generally smaller than in the case of the oscillations of the first type under the same conditions. The dependence of ΔU upon the pressure is appreciable, as can be seen from Fig. 1. The average voltage on the tube was 22—23 V. Striking is the large value of the oscillations at optimum pressure—13 V, *i. e.* 60 per cent of the constant component. With a rise in current strength, ΔU falls, just as in the case of the oscillations of the first type. The dependence of ΔU upon the length of the discharge gap is shown in Fig. 2 (upper curve); below is shown the constant component of the voltage under the same conditions. Remarkable is the total absence of oscillations when $l < 30$ mm, and the simultaneous fall in voltage on the tube to the value of the cathode drop ($\sim$10 V). Herewith the discharge likewise changes in character: all light emission, aside from the cathode spot, vanishes; a dark space fills the entire tube above the cathode. This repeatedly checked experiment shows that the cathode regions of the discharge, up to the Faraday dark space inclusive, are not the source of the oscillations of the second type (and also the third, see below).

3. Regular Oscillations of Low Radio-frequency. These oscillations appear only in the case of an anchored cathode spot and at low pressure (not above several μ Hg) in a certain definite region of values of the cur-

* *i. e.* the difference between the extreme values of the voltage: $\Delta U = U_{max} - U_{min}$. If the oscillations were sinusoidal with a constant component, then ΔU would be equal to double the amplitude of the sinusoid.

rent i. At a certain value $i=i_{res}$. the amplitude of these oscillations is the largest (for the typical relation see Fig. 3) and the phase is highly stable, so that the oscillogram of the oscillations may be stabilized on the screen of the oscillograph and photographed. To both sides of i_{res} the regular oscillations diminish, and a background of chaotic oscillations of the second type appears; finally, when $i \gg i_{res}$ or $i \ll i_{res}$ the regular oscillations vanish completely and are replaced by chaotic ones. This phenomenon may be characterized as a sort of «resonance» of the discharge.

The resonance current i_{res} depends upon the pressure, as can be seen from the following table:

p (μ Hg)	0.5	1.0	1.2	1.43	2	3.2
i_{res} (A)	7.5	6.8	5.5	5.0	4.2	4.0

The shape of the oscillations is varied, generally non-sinusoidal and abounding in harmonics. The fundamental frequency f varies between the limits of 20 000 and 100 000 cycles depending upon the pressure, current strength and length of discharge: f is a decreasing function of p. With a change in l the frequency at first abruptly increases until $l=6$ cm, then slowly decreases, changing little later on.

The amplitude of the oscillations of the third type is a complex function of l, but when $l \leq 20$ mm the regular oscillations vanish just as in the case of the oscillations of the second type.

4. Regular Oscillations of Sound Frequency. These oscillations are also observed in the case of anchoring of the cathode spot in the pressure region $p<15$ μ, when oscillations of radio-frequency are practically undetectable. The phenomenon of «resonance» current strength is likewise very manifest, but the region of resonance is much narrower, and with a rise in pressure i_{res} increases. The shape of the oscillations is varied; at higher pressures nearly sinusoidal oscillations appear. The frequencies lie within the limits 300—3000 cycles and hence may be easily detected with a telephone.

§ 3. A comparison of the data shows that the oscillations of the first type originate from the movement of the cathode spot, while the oscillations of the second, third and fourth types cannot possibly arise in the cathode regions of the discharge. Their source may lie either in the positive column or in the anode region. To solve this problem a tube was prepared with two wire probes: one, S_1, 1 mm from the anode, and the other, S_2, in the middle of the tube. Then the potentials of S_1, S_2 and the anode A with respect to the cathode K, and also the potentials of S_1, S_2 and the cathode K with respect to the anode A, were recorded with an oscillograph.

In the case of the oscillations of the second type it was found that if the oscillations of the voltage $K—A$ are taken as 100 per cent, then ΔU is equal only to 30 per cent in the segment $K—S_1$ and is also $\sim$30 per cent in the segment $K—S_2$. Hence, 70 per cent of the voltage oscillations arise right at the anode in the segment $S—A_1$, 1 mm in length. That this is so follows from the observations of the oscillations with respect to the anode: the segments $S_1—A$ and $S_2—A$ furnish practically as much as the entire tube. In the case of oscillations of the third type 19 per cent of the full amplitude of the oscillations were found in the $K—S_2$ segment and 25 per cent in the $K—S_1$ segment, *i. e.* a still more striking picture than above.

Finally, in the case of oscillations of the fourth type, the following picture for ΔU was found

	$K—S_2$	$K—S_1$	$K—A$	$S_1—A$	$S_2—A$
ΔU	0	0	100%	100%	100%

i. e. the complete concentration of all the oscillations in the anode region.

Thus, it has been proved that the region of anode drop is the point of origination of the oscillations in the case of an anchored cathode spot.

A number of experiments were set up in which the anode region of the discharge was acted upon in various ways, and the effect on the oscillation of the voltage studied.

a) Various potentials (+ and —) were impressed on the probes with respect to the anode. Probe S_1, in the case of a positive potential, intensified the oscillations up to 30 per cent; probe S_2, when the probe current was exactly the same, gave an intensification of not more than 7—9 per cent. Negative potentials of up to—3000 V had no effect whatever.

b) An auxiliary incandescent cathode was sealed into a special side-arm of the discharge tube at a level with the anode; the electrons it emitted were accelerated in the direction of the anode, where they caused additional ionization of the gas. Under «resonance» conditions the electron beam of 100 mA diminished the amplitude of the regular oscillations by $\sim$ 12 per cent and slightly changed their shape. At a distance from the resonance we observed not only a decrease in amplitude but transition of the oscillations into the chaotic type 2. The oscillations of the second type were weakened by the electron beam by $\sim$8 per cent, not changing their character.

c) A jet of mercury vapour was blown in through a special nozzle directly in front of the anode. A comparatively weak jet diminished the amplitude of the oscillations of the third type by 9—10 per cent and made their shape more complex. When the jet was strong, the regular oscillations changed into chaotic, while their amplitude diminished fourfold.

It seems probable that the source of the oscillations is the periodic process in the anode region, advanced by Pupp [5] to explain the moving sheets: a change in the anode drop evokes a change in the velocity of the electrons and the intensity of ionization of the gas, owing to which the space charge of the anode changes, in its turn affecting the anode drop in the opposite direction.

The existence of a «resonance» current strength in the discharge may apparently be explained by the stabilization and intensification of the oscillations in the anode region induced by the oscillations of the plasma of the discharge, whose possibility was theoretically investigated by one of us [9].

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PHYSICS

ON STABILITY OF NEIGHBOURING ISOBARS

By L. I. RUSINOV and J. M. IGELNITZKY

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At present three isobaric pairs are known which show no observable beta-radioactivity of either positive or negative sign. These are Cd_{48}^{113}—In_{49}^{113}, In_{49}^{115}—Sn_{50}^{115} and Sb_{51}^{123}—Te_{52}^{123}.

On the other hand, according to the theory of radioactive transmutations and also on general considerations, when energy is taken into account, one should expect radioactive disintegration of one of the two neighbouring isobar-nuclei (1).

The necessary condition for positive or negative electrons to be emitted by radioactive nuclei is that the difference in mass should be greater than the sum of the masses of an electron and a neutrino. On the other hand, radioactivity due to the capture of an orbital K-electron, can take place when the difference between the masses of the respective nuclei is only greater than the mass of a neutrino (the energy of ionization of the K-level in the atom must, of course, be taken into consideration).

Since neither electronic nor positronic decay has been detected in the above-mentioned neighbouring isobars, it may be supposed that in their case the radioactive transformations proceed through capture of a K-electron by the nucleus.

The most accurate measurements of the lifetime of neighbouring isobars with respect to the K-capture phenomenon have been carried out by Zingg(2). From the X-ray emission attending the phenomenon of K-capture, which he measured by means of an ordinary Geiger-Müller counter filled with air, Zingg determined the lower limit of half-life for isobaric nuclei as $T \sim 10^{12}$ years.

The present authors also have made some experiments aimed at ascertaining the stability of neighbouring isobars towards processes of capture of an orbital electron. They used a technique which is very sensitive for measuring soft gamma-quanta, and for the description of which the reader is referred to an earlier paper(3).

The measurements were made in the same order as in the experiments on the detection of a long-lived nuclear isomery. We tried to detect K-ray quanta which, in case the process of absorption of a K-electron by the nucleus had taken place, should be emitted by the neighbouring isobars with high atomic numbers. The X-rays sought-for were not observed in these experiments. For the limit of the half-life T the following data were obtained

Element	In_{49}^{113}	Sn_{50}^{115}	Te_{52}^{123}
T, years	$> 1 \cdot 10^{14}$	$> 1 \cdot 10^{13}$	$> 2 \cdot 10^{14}$

To account for the results obtained one may suppose that owing to a high forbiddance (in the case of In^{113}—Cd^{113}, for instance, $\Delta l = 4$) the half-periods of disintegration with respect to K-capture (or β-disintegration) in the isobars investigated are actually higher than the limits found in the experiment. Possibly, these neighbouring isobars are stable in general on account of the energy relations pointed out above.

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PHYSICAL CHEMISTRY

A STUDY OF THE ELASTO-PLASTIC PROPERTIES AND THE THIXOTROPY OF DISPERSE SYSTEMS

By S. J. VEILER and P. A. REHBINDER, Corresponding Member of the Academy

Structural-mechanical properties—elasticity under shear, elasto-kinetic and elasto-plastic (relaxation) properties—are fundamental characteristics of structure formation in disperse systems and of related phenomena, such as gelation, thixotropy, *etc.*

In the elasto-plastic properties of structured systems lies the reason for the anomalies observed in their viscosity. Therefore the study of the elasto-plastic characteristics in disperse systems may be recommended in place of the viscosity measurements usually employed in colloidal chemistry.

The elastic properties of structural liquids (hydrosols of gelatin) were first measured by Schwedoff (1) with a torsion apparatus having coaxial cylinders (the relaxation of elastic stresses in sols was studied and the rigidity modulus determined). Further research in this direction was carried out by Rohloff and Shinjo(2), Hatschek and Jane (3), Trouton and Andrews (4), Arcay and Etienne (5) and others. This method has a number of shortcomings, namely, the influence of contact (edge) effects, the intricacy of the apparatus both as regards manufacture and mounting, and the difficulties in selecting a torsion thread, whose rigidity modulus must correspond to the modulus of the given system.

To determine the viscosity and ultimate shearing stress in highly viscous systems (bitumens, soap, dyes and others) methods involving the longitudinal displacement of coaxial cylinders were employed (Segel (6), Tamman (7), Pochettino (8)). Pochettino measured the axial displacement of a cylindrical rod in a coaxial tube containing the liquid under investigation. An essential shortcoming of this type of apparatus is that sliding is apt to arise at high shear stresses. Later Tolstoy (9), working in Volarovich's laboratory, employed similar devices in which sliding was eliminated by providing the rod with a sharp screw thread on its lateral surface. Volarovich and Tolstoy (10) resorted to stretching out the rod (a kind of «screw», which had previously been soldered or «screwed» into the system) by means of a spring dynamometer of 1.7 - 15 kg. In an analogous device Mokievsky, in Rehbinder's laboratory (11), had a stage lowered by a screw, which ensured small and constant velocity of deformation in the system.

On the other hand, in measuring high viscosities a method based on the displacement of a plate in its own plane was employed (Nutting (12)). Weinberg (13) determined the viscosity of tar, resins, bitumens, and other materials from the displacement of the respective material, effected by means of a plate under constant stress. His apparatus was improved by Dudetsky (14), who made the plate move under the action of a constant force between two layers of the material investigated, and measured the resultant velocity of shear. Similar in principle is the band viscometer of Wachholtz and Asbeck (15).

Measurement of the ultimate (static) shearing stress in clay suspensions is effected in the American oil industry with a so-called shearometer ([16]). This apparatus consists of an oblong ebonite plate ($158 \times 25 \times 0.8$ mm), which is lowered vertically into the liquid. The ultimate shearing stress is determined from the depth of immersion. The shortcoming of the method lies in the inconstancy of the surface area wetted, the presence of edge phenomena (surface tension) and the inaccuracy of the measurements.

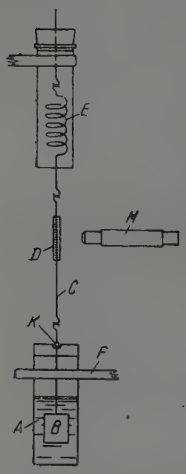

Fig. 1. *A*, vessel with system under investigation; *B*, plate; *C*, glass thread; *D*, photomicrograph scale; *E*, spring; *F*, disk with sockets for vessels; *K*, arrester of plate; *M*, microscope

To determine the elastic properties of thixotropic suspensions and colloidal solutions we have worked out an apparatus (Fig. 1) based on the principle of tangential displacement of a plate within the system. A rectangular thin plate of mica attached to a glass thread suspended from a quartz or glass spring, like in the McBain balance ([17]), is completely submerged in the suspension under insvestigation. By means of a motor the support of the vessel containing the suspension—an oblong pan or a beaker—is lowered at a constant speed (in our experiments 3.3 μ/sec). through a reductor. The constant speed of deformation can easily be adjusted to make the shear and ultimate (critical) stress diagrams independent of the velocity. The displacements of the plate and the extension of the spring are read with the aid of a horizontal microscope on a microscale fastened along the glass thread. By this method stress—strain curves are readily obtained (Fig. 2).

To establish the possibility of measuring absolute values of the ultimate shearing stress as a definite physical quantity by this method, it was necessary to prove its independence of the dimensions of the plate and the shape of the vessel. Plates of copper and aluminium foil and of mica with rough surfaces were tested. The invariability of the ultimate shearing stress for an 18 per cent clay suspension is shown in Table 1.

The suspensions were prepared by Zurinov's method ([18]) with floridine («colloidal») clays from the Chistopol deposit in the Tatar Autonomus Republic. According to our measurements, the degree of dispersion of these clays is characterized by the

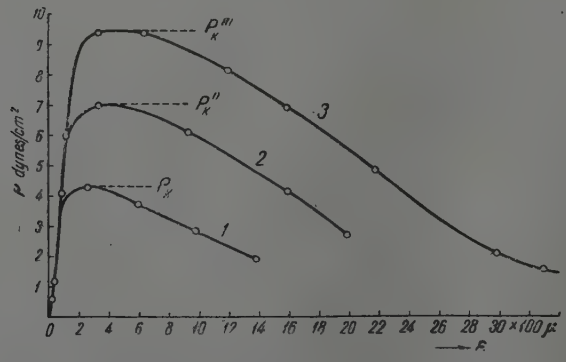

Fig. 2. *1*, after 1 hr.; *2*, after 4 hrs; *3*, after 10 hrs

following data: with respect to mass 81 per cent of the particles are less than 5 μ in diameter, and 15 vary from 5 to 10 μ. The suspensions were prepared by grinding up the clay in a mortar and adding a 1 per cent aqueous solution of sodium carbo-

Table 1

Dimensions of plate			Area $2S$, in cm²	Ultimate stress in in extending the spring F_m dynes	Ultimate shearing stress $P_m = \frac{F_m}{2S}$ dyne/cm²	Vessel
width, in cm	length, in cm	thickness, in μ				
2.5	3.0	50	15	163	10.88	Cylindrical beaker 8 cm in diameter
3.0	4.0	60	24	261	10.88	A pan with plane-parallel walls, 10 × 10 × 5 cm
4.0	5.0	65	40	424	10.60	
5.0	7.0	60	70	758	10.83	
7.0	7.9	60	111	1214	10.94	

nate (Na_2CO_3) by crops until a thick paste was obtained; then the paste was diluted by the same solution to the desired concentration.

To illustrate the application of the apparatus we give in Table 2 data of the thixotropy of an 18 per cent clay suspension obtained according to the method employed in our laboratory ([19]) for measuring the kinetics of thixotropic gelation $P_m = f(\tau)$—ultimate shearing stress as a function of restoration (structure formation) time τ—at $t = 20°$ C. The area of the plate, $2S$, was 40 cm².

Table 2

Kinetics of Thixotropic Gelation $P_m = f(\tau)$

Time of restoration of structure after complete failure in min.	Ultimate shearing stress P_m dyne/cm²	Time of restoration of structure after complete failure, in hrs.	Ultimate shearing stress P_m dyne/cm²
2	0.0140	1	4.30
3	0.0350	2	5.40
5	0.0700	4	7.00
7	0.140	10	9.30
11	0.350	18	10.5
15	0.700	24	10.8
23	2.00	28	11.0

Developing previous researches by our laboratory ([20]), J. Aron succeeded in revealing with our method the appearance of the first traces of structure in diluted solutions of high polymers under the influence of active fillers.

Dumansky ([21]), employing our method in the investigation of the elastic properties of gelatin solutions, established the complete invariability of the shear modulus (rigidity) within a broad range of plate areas and for various container dimensions. In this research he employed a special container with a movable inner wall.

The developed method makes it possible to measure the ultimate shearing stress, the modulus of elasticity, relaxation, and also the effective viscosity at various velocities of deformation in a broad range of consistency from weakly structured sols to solid-like systems with a highly stable structure. The apparatus is highly sensitive and makes it possible to employ a large shear area with very small plate weight, and also strictly control the temperature conditions of the experiment while carrying out a series of determinations.

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CHIMIE PHYSIQUE

SUR LA DÉTERMINATION DE LA COMPOSITION D'EUTEXIE BINAIRE SE DÉGAGEANT DANS LES SYSTÈMES TERNAIRES A UNE TEMPÉRATURE DONNÉE

Par O. S. IVANOV

(Présenté par N. T. Gudzov, de l'Académie, le 5. X. 1945)

Il est admis dans les travaux [1-3] sur les systèmes ternaires que la composition d'eutexie binaire se dégageant d'une solution à une température fixée est déterminée par le point d'intersection de la tangente à la courbe $e_1 e_2$ (fig. 1) avec la conode qui joint les compositions de deux phases solides en équilibre avec le liquide en question.

Notre communication a pour objet de démontrer que cette méthode n'est vérifiée que dans des cas particuliers, mais qu'en général elle doit être remplacée par une autre.

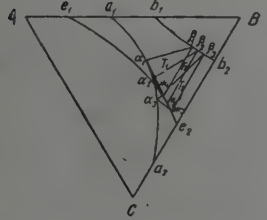

Fig. 1. Le procédé généralement admis de définition de la composition d'eutexie binaire qui se dégage à la température considérée du liquide du système ternaire

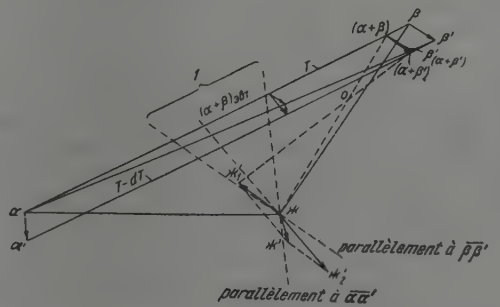

Fig. 2. La définition de l'intervalle *1*, où le changement de la composition d'eutexie binaire a lieu; celle-ci est dégagée à la température considérée du liquide du système ternaire et dépend de la composition moyenne de l'alliage

Analysons en détails les changements qui ont lieu dans un système au moment où l'eutexie binaire se dégage dans un alliage ternaire à trois constituants.

L'alliage dont la composition est exprimée par le point o à la température T consiste du liquide de la composition $\varkappa$ et de deux solutions solides dont les compositions sont α et β. Admettons que la température baisse en ce moment à une valeur quelconque dT. Les compositions de toutes les phases doivent alors varier le long des courbes $a_1 a_2$, $b_1 b_2$ et e_1, e_2 et à une température nouvelle $T-dT$ elles seraient exprimées par les points α', β' et $\varkappa'$.

En rapport avec notre problème déterminons la composition d'eutexie binaire qui se dégage en présence d'un abaissement de température. Rappelons que nous examinons les états d'équilibre exprimés par un diagramme stable; par conséquent, toutes les phases se trouvent en équilibre réciproque et chacune d'elles est tout à fait homogène à tout moment donné. Le changement du système lui-même est déterminé comme une différence entre deux états: l'état initial et l'état final.

Tant qu'il s'agit d'un diagramme d'équilibre il est naturel de ne pas considérer ici le mécanisme du changement. Dans notre cas le dégagement d'eutexie binaire est compris comme l'accroissement quantitatif des phases solides aux dépens du liquide, sans considérer si la formation des cristaux nouveaux a lieu ou si les phases solides se déposent à la surface des cristaux préalablement formés (cristaux primaires).

La partie du diagramme d'état du mélange qui nous intéresse est représentée par la figure 1, en élargi—sur la figure 2. dT étant infiniment petit, les portions des courbes, a_1a_2, b_1b_2 et e_1e_2 se transforment en droites $\alpha\alpha'$, $\beta\beta'$ et $\mathit{ж}\mathit{ж}'$.

Tant qu'il s'agit d'un changement de T et, par conséquent, d'un changement du système aussi petit que nous le voulons, nous pouvons diviser ce changement en deux parties tout en gardant l'équilibre constant du système: le changement de la composition des phases et le changement de leurs quantités, c'est-à-dire la cristallisation elle-même d'eutexie binaire. Commençons par l'analyse de la composition des phases.

Quand un abaissement de température de valeur dT a lieu, la composition des phases solides atteint le point $(\alpha+\beta)'$, grâce au changement des concentrations α et β des solutions solides.

Le mode de déterminer le point $(\alpha+\beta)'$ est basé sur le fait que les segments suivants sont parallèles

$$(\alpha+\beta)\overline{\beta'}_{(\alpha+\beta)} \parallel \beta\beta', \quad \overline{\beta'_{(\alpha+\beta)}(\alpha+\beta)'} \parallel \overline{\alpha\alpha'}$$

Ils le sont en raison des relations suivantes qui ont lieu lorsque la concentration des solutions solides varie en même temps que leur quantité reste constante:

$$\frac{m_\alpha}{m_\beta} = \frac{\overline{(\alpha+\beta)\beta}}{\overline{\alpha(\alpha+\beta)}} = \frac{m_\alpha}{m_\beta} = \frac{\overline{\beta'_{(\alpha+\beta)}\beta'}}{\overline{\alpha'_{(\alpha+\beta)}}} = \frac{m_{\alpha'}}{m_{\beta'}} = \frac{\overline{(\alpha+\beta)'\beta'}}{\overline{\alpha'(\alpha+\beta)'}}$$

Tant que la composition moyenne de l'alliage est fixée au point o, le déplacement de la composition moyenne des phases solides du point $(\alpha+\beta)$ au point $(\alpha+\beta)'$ devrait transférer la composition du liquide en point $\mathit{ж}$. La position de ce point est détermirée par le fait que les quantités de la phase liquide et de deux phases solides restent en attendant invariables. Par suite

$$\frac{m_\alpha+m_\beta}{m_{\mathit{ж}}} = \frac{\overline{\mathit{ж}o}}{\overline{o(\alpha+\beta)}} = \frac{m_{\alpha'}+m_{\beta'}}{m_{\mathit{ж}_1}} = \frac{\overline{\mathit{ж}'_1 o}}{\overline{o(\alpha+\beta)'}}$$

Les triangles $o(\alpha+\beta)(\alpha+\beta)'$ et $o\mathit{ж}\mathit{ж}'$ sont semblables, et $\mathit{ж}\mathit{ж}'_1 \parallel (\alpha+\beta)(\alpha+\beta)'$.

Mais conformément au diagramme d'état du mélange, lorsque la température baisse jusqu'à $T-dT$, la composition du liquide doit être $\mathit{ж}'$, ce qui aura lieu en effet, grâce au dégagement de l'eutexie binaire. La composition du liquide se déplacera ainsi du point $\mathit{ж}$ au point $\mathit{ж}'$ sous l'influence de deux processus simultanés: celui du changement des phases solides et celui du dégagement d'eutexie binaire. Sous l'influence du premier processus la composition du liquide se déplacera lentement vers le point $\mathit{ж}'_1$, et sous l'influence du deuxième processus — vers le point $\mathit{ж}'_2$, de façon qu'en résultat de l'addition de ces deux déplacements la composition du liquide arrive au point $\mathit{ж}'$, qui se trouve sur la courbe e_1e_2. On détermine le point $\mathit{ж}'_2$ comme la terminaison du vecteur obtenu en soustrayant le vecteur $\overrightarrow{\mathit{ж}\mathit{ж}'_1}$ du vecteur $\overrightarrow{\mathit{ж}\mathit{ж}'}$.

Par suite, la composition de l'eutexie binaire se dégageant à la température T est déterminée par le point où la conode $\alpha\beta$ se rencontre avec le prolongement du segment $\overline{ж ж'_2}$ et non point, comme il est souvent admis, avec le prolongement du segment $\overline{ж ж'}$, qui est une tangente à la courbe e_1e_2 au point $ж$.

Les résultats obtenus nous amènent aux conclusions suivantes:

1. La direction $ж\ (\alpha+\beta)_{eu}$ est donneé par le vecteur $\overrightarrow{ж ж'_2}$, qui, comme il est déjà noté, présente la différence entre les vecteurs $\overrightarrow{ж ж'}$ et $\overrightarrow{ж ж'_1}$. A la température considérée T le vecteur $\overrightarrow{ж ж'}$ est le même pour les alliages dont la composition est comprise dans le triangle à trois phases $\alpha\beta ж$. Au contraire, le vecteur $\overrightarrow{ж ж'_1}$ dépend, en ce qui concerne sa valeur et sa direction, de la composition moyenne des alliages. Par conséquent, la position du vecteur $\overrightarrow{ж ж'_2}$, ainsi que la direction $ж\ (\alpha+\beta)_{eu}$, dépendent de même de la composition moyenne de l'alliage.

Par suite, la composition de l'eutexie binaire, qui se dégage à la température T et qui est fixée par le point $(\alpha+\beta)_{eu}$, dépend de la composition moyenne de l'alliage. Dans l'eutexie binaire, formée à la même température (l'intervalle de températures), le rapport $\alpha:\beta$ doit varier, ce qui dépend de la composition moyenne de l'alliage. Contrairement à cet avis et d'accord avec celui qui est exprimé dans les travaux ci-dessus mentionnés, le rapport $\alpha:\beta$ d'eutexie binaire, formée à une température définie T, devrait être indépendant de la composition de l'alliage. Soulignons que cette conception n'est juste que dans les cas particuliers, notamment, lorsque la composition des phases solides ne change pas pendant l'abaissement de la température. Cela a lieu, par exemple, pendant le dégagement de l'eutexie binaire, formée des constituants pures, ou lorsque les courbes a_1a_2, b_1b_2 et e_1e_2 sont réciproquement parallèles, à la température considérée.

Il y a trois procédés de déterminer la composition d'eutexie binaire dégagée en fixant les points d'intersection de la tangente à la courbe e_1e_2 avec la conode $\alpha\beta$: 1) au début de la cristallisation des alliages dont la composition moyenne se trouve sur la courbe e_1e_2; 2) à la température T, donnée pour tous les alliages, lorsque les tangentes aux trois courbes a_1a_2, b_1b_2, et e_1e_2 sont à cette température parallèles; et 3) pour les alliages dont la composition se trouve sur la ligne droite joignant la composition du liquide $ж$ avec le point $(\alpha+\beta)$ lorsque le segment $(\alpha+\beta)\overline{(\alpha+\beta)'}$ est parallèle à la tangente de la courbe e_1e_2 au point $ж$.

2. La position du point $(\alpha+\beta)_{eu}$ par rapport aux points α et β (fig. 2) nous permet de définir le caractère de la réaction de cristallisation: est-ce le dégagement d'eutexie binaire ou la réaction pereutectique $ж+\beta\rightarrow\alpha$ qui a lieu ici. Dans le premier cas le point $(\alpha+\beta)_{eu}$ se trouve entre les points α et β, dans le second cas — à gauche du point α. Comme il a été démontré plus haut, la position du point $(\alpha+\beta)_{eu}$ dépend de la composition moyenne de l'alliage. C'est pourquoi dans les différents alliages du système la transition de la réaction pereutectique $ж+\beta\rightarrow\alpha$ en réaction eutectique $ж\rightarrow\alpha+\beta$ s'effectue à différentes températures et dépend de la composition moyenne de l'alliage, ce qui est représenté par la figure 3. La conception généralement admise que la substitution de la réaction pereutectique par la réaction eutectique a lieu dans tous les alliages à la même température, est inexacte (1).

Cette transition est représentée sur la figure 3 par une allure naturelle des courbes a_1a_2, b_1b_2 et e_1e_2. La transformation du triangle pereutectique en triangle eutectique (substitution de la réaction pereutectique $ж+\beta\rightarrow\alpha$ par la réaction eutectique $ж\rightarrow\alpha+\beta$) commence dans ces conditions à la température T_1 par le sommet α_1, au moment où la tangente courbe a_1a_2 au point α_1 devient parallèle à la conode $\alpha_1ж_1$.

En analysant la figure 2, il est facile de comprendre qu'à ce moment pour l'alliage de la composition moyenne α_1 le point $(\alpha+\beta)_{eu}$, se déplaçant de gauche à droite, passe par le point α_1, ce qui correspond à la transition de la réaction pereutectique en réaction eutectique. Pendant l'abaissement ultérieur de la température le domaine des alliages à trois phases, où la réaction eutectique a lieu, s'étend du sommet α vers les sommets $\mathcal{M}$ et β. A la température T_2 la limite séparant la partie eutectique du triangle de la partie pereutectique passe par le sommet $\mathcal{M}_2$. A ce moment la tangente à la courbe p_1e_2 devient parallèle à la conode $\alpha_2\mathcal{M}_2$. Enfin, à la température T_3 tout le triangle à trois phases devient eutectique. A ce moment la tangente à la courbe b_1b_2 au point β_3 devient parallèle à la conode $\alpha_3\mathcal{M}_3$. En général, la limite qui sépare la partie pereutectique du triangle de la partie eutectique passe par ses sommets suivant l'ordre dans lequel les tangentes aux courbes a_1a_2, b_1b_2 et p_1e_2 en ces sommets deviennent parallèles à la conode $\alpha\mathcal{M}$.

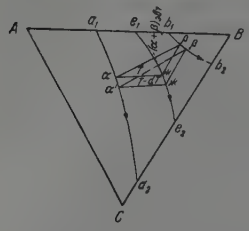

Fig. 3. L'influence de la composition de l'alliage sur la température de la transition de cristallisation de pereutexie binaire en cristallisation d'eutexie binaire

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GEOCHEMISTRY

RARE EARTHS IN MINERALS

By I. B. BOROVSKY and V. I. GERASIMOVSKY

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1. The elements of the group of rare earths, from La to Lu, with atomic numbers from 57 to 71 inclusive, constitute one of the most interesting series within Mendeleev's periodic table.

The specific physical-chemical properties of the rare earth elements (1), their co-occurrence under natural conditions (in minerals, ores and soils) are associated with the specific features of the electronic structure of the atoms of these elements: the gradual completion of the 4*f*- and 5*d*-concentric orbits while the 6 *s*-orbits are filled up (2).

The quantitative identification of the rare earth elements by the routine methods of analysis involves considerable difficulties, and the obtaining of pure samples of some of them is a rather cumbersome procedure. This is why their distribution in the Earth crust has been but poorly investigated.

2. The actual concept of the distribution of rare earths in the Earth crust, and of the quantitative relations between them, is largely based upon data by Goldschmidt and Tomassen (3).

One of the present authors (4) undertook to analyse quantitatively with the aid of the X-ray spectroscopic method the rare earths present in minerals occurring in the USSR. The experimental data obtained made him arrive at conclusions somewhat different from those generally adopted with regard to the quantitative relations between the elements of this group and the dependence of their distribution in the various rocks of the Earth crust upon the nature of these rocks.

Quantitative X-ray spectrum analysis of the elements of the TR group was carried out with the aid of spectrographs with bent crystals, according to Johann and Cauchoix, by the method of internal standards. Experimentally obtained transitional coefficients between the relative intensities of the lines and the content of the elements by weight were used (5).

3. Table 1 contains the results of a quantitative analysis of 23 minerals.

The average relative content of atoms of the elements of the cerium group, as borne out by this table, is as follows: 57 La : 58 Ce : 59 Pr : 60 Nd = 0.66 : 1 : : 0.20 : 0.41, *i. e.* Ce > La > Nd > Pr > Sm (Sm has been identified for five minerals).

Goldschmidt's and Tomassen's data which are commonly adopted in geological and geochemical literature, lead to the following atomic ratio: 57 La: 58 Ce: 59 Pr: :60 Nd = 0.23 : 1 : 0.16 : 0.58, *i. e.* Ce > Nd > La > Pr.

Thus, according to Goldschmidt La : Nd is 0.4, whereas from our data it is 1.6.

Among all the samples analysed the correlation between the individual elements of the cerium group is found to coincide with that reported by Goldschmidt only in two samples of apatites, from the Volkov deposits (the Urals), and from the Vakis-Djvari deposits (West Georgia), both of which correspond to basic rocks.

Table 1

Name of sample and of rock to which it is confined	Ce	Rd/Ce	La/Ce	Pr/Ce	ΣTR, %
Apatite from Volkovsky deposit the Urals (gabbro)	1	0.90	0.50	0.40	~ 1
Apatite Kuftuay, Lovozersky tundras (nephelite syenites)	1	0.41	0.52	0.15	~ 1
Grey apatite from the Vakis-Djvari deposit, West Georgia (gabbro)	1	0.65	0.45	0.22	~ 1
Loparite from urtite of Mt. Vavnbed (nephelite syenites)	1	0.30	0.60	0.20	~ 30
Loparite from urtite of Mt. Eingporr (nephelite syenites)	1	0.40	0.60	0.20	~ 30
Loparite from luyavrite of Mt. Vavnbed, Lovozersky tundras (nephelite syenites)	1	0.35	0.60	0.20	~ 30
Loparite from Mt. Ninchurt, Lovozersky tundras (nephelite syenites)	1	0.32	0.41	0.12	~ 30
Loparite from Mt. Aluayv, Lovozersky tundras (nephelite syenites)	1	0.34	0.62	0.17	~ 30
Monazite from the Kounrad deposit, North-western near-Balkhash (granites)	1	0.24	1.10	0.30	~ 60
Monazite from the Kounrad deposit, North-western near-Balkhash (granites)	1	0.24	0.83	0.14	~ 60
Monazite of the Taraka deposite (granites)	1	0.44	0.51	0.14	~ 60
Monazite Sukhoy Padunok of the Tarak deposit (granites)	1	0.44	0.64	0.19	~ 60
Monazite from pegmatite, Elizavetinsky mine, South Urals (granites)	1	0.43	0.63	0.19	~ 60
Monazite from a pegmatite of the Uruskiskan river, South Urals, Mironovsky mine (granites)	1	0.50	0.70	0.20	~ 60
Granite monazite, Uruskiskan river, settlement Andyrmensky (granites)	1	0.44	0.90	0.25	~ 60
Parisite from Mariupol (nephelite syenites)	1	0.36	0.73	0.16	~ 60
Parichlore from Kurochkin log, the Vishnevye mounts of the Urals (nephelite syenites)	1	0.28	0.74	0.23	~ 4
Nordite from Mt. Changusuay, Lovozersky tundras (nephelite syenites)	1	0.29	1.05	0.20	~ 18
Orphite, Lovozersky tundras (nephelite syenites)	1	0.45	0.52	0.18	~ 20
Orthite from the Baby-djid river, North Kirghizia (granites)	1	0.34	0.70	0.18	~ 23
Soquan clay, Moscow province	1	0.49	0.46	0.19	~ 0.01
Palaeozoic shale (Europe) (6)	1	0.30	0.48	0.14	~0,047
Mesozoic shale (Japan) (6)	1	0.43	0.46	0.12	~ 0.02
Mesozoic shale (Japan) (6)	1	0.40	0.29	0.11	~ 0.02

The mean ratio Ce:Nd:La:Pr = 1 : 0.41 : 0.66 : 0.20. If we take $\Sigma\ Ce_2O_3 = 100$ per cent (minus the yttrium group), the following content of the individual elements of the cerium group will be obtained (resulting from a total of 24 analyses) Ce_2O_3, 43.0%; Nd_2O_3, 17.7%; La_2O_3, 28.5%; Pr_2O_3, 8.6%; Sm_2O, 2.2 %.

From Table 1 it may easily be inferred that in minerals associated both with basic rocks and with granites and nepheline syenites, the ratio of the elements of the cerium group does but little deviate from the average; and in two minerals only, *viz.* the monazite from the north-west near-Balkhash and the nordite from Lovozersky tundras do we find La > Ce.

Table 2 illustrates the quantitative relation between the elements of the yttrium group, obtained from 8 analyses of minerals. A comparison of the data contained in this table with Goldshmidt's data leads to the following results:

According to Goldschmidt: 39 Y>64 Gd=66 Dy=70 Yb>68 Er> 71 Lu> >65 Tb=67 Ho=69 Tu>63 Eu.

According to Borovsky: 39 Y>64 Gd>66 Dy>68 Er> 70 Yb> 63 Eu> >65 Tb≥67 Ho≥69 Tu≥71 Lu.

Table 2

Semi-quantitatve X-ray-spectrum Analysis of Elements of the Yttrium Group (in per cent)

Name and origin of the sample	Eu	Gd	Tb	Dy	Ho	Er	Tu	Yb	Lu	Y
Palaeozoic shale (Europe)	2	14	2	10	3	6	0.6	6	2	35
Palaeozoic shale (Japan)	3.5	10	25	9	2	4	0.4	3	1	64
Liparite, Kola peninsula	7	45	5	25	2	5	2	3	—	10
Monazite, Taraka . . .	1	15	2	10	1	5	—	4	1	64
Monasite, Baschelok . .	—1	14	2	9	1	6	1	2	2	62
Monazite, Sukhoy padunok	—	15	2	10	1	4	0.6	3	0.3	62
Monazite, Baley	1.5	16	2	13	1.5	4	1	4	—	57

The most essential in the rearrangement of the sequence of elements is according to our data the mutual transposition of 63 Eu and 71 Lu, the former being shifted from its tenth place to the sixth place.

Our data demonstrate, moreover, a regular decrease in the relative quantities of elements of the yttrium group, accompanied by an increase both in even and odd atomic numbers. Especially overestimated has been by Goldschmidt the rôle of Y within this group of elements.

No less striking appears the difference between our data and those obtained by Goldschmidt if we are to compare the relative contents of elements of the cerium and yttrium groups. The highest content in the yttrium group has been found in three apatites, two of which are confined to basic rocks, while the third is associated with nephelite syenites.

The relative content in the elements of this group has been found to vary but very little in all the eight analyses. In Table 3 is to be found the content of rare earths in three apatites and one loparite sample (ΣTR being put at 100 per cent).

The quantitative relations between the individual elements both of the cerium and yttrium group of rare earths are found to vary within a rather narrow range in minerals differing as to their genesis and confined to different rocks (basic, granites or nephelite syenites). The ΣCe_2O_3 to ΣY_2O_3 ratio is subject to no important variation either, being the highest in apatites.

If we are to judge by what has been reported above for minerals of the USSR, it may well be assumed that a sharp differentiation between the elements of rocks and those of nephelite syenites is scarcely like to take place.

The ratios of elements belonging to the group of rare earths, as suggested by the present author, vary materially from those given by Goldschmidt; they are, however, in fair agreement with more recent individual measurements made by some other students (6).

Table 3

	Apatite, Vakis-Djvari deposit, West Georgia, in %	Apatite, Volkovsky deposit, Urals, in %	Loparite, from Mt. Vavnbed, in %	Apatite, from Kirovsky mine, Kola peninsula, in %
57 La_2O_3 . . .	17	15	26	27
58 Ce_2O_3 . . .	35	30	44	34
59 Pr_2O_3 . . .	8	11	9	7
60 Nd_2O_3 . . .	23	25	17	16
62 Sm_2O_3 . . .	2	2	2	2
63 Eu_2O_3 . . .	1	0.5	0.05	0.8
64 Gd_2O_3 . . .	4	5	0.6	5
65 Tb_2O_3 . . .	0.6	0.7	0.5	0.5
66 Dy_2O_3 . . .	3	4	0.3	3
67 Ho_2O_3 . . .	0.3	0.2	0.02	?
68 Er_2O_3 . . .	2	2	0.05	1
70 Yb_2O_3 . . .	1	0.8	0.03	?
71 Lu_2O_3 . . .	0.1	0.1	—	—
39 Y_2O_3 . . .	3	4	0.1	3
69 Tu_2O_3 . . .	—	—	0.02	—

The validity of the ratios obtained for the elements of the yttrium group does not seem to be affected by the fact that our results are lacking analytical data for yttrium minerals; nor do we feel obliged to restrain the scope of our generalizations, in so far as apatite, monazite, orthite, loparite are the most widespread minerals containing rare earths.

It should be pointed out, moreover, that the relations recorded for the elements within the TR group contained in sedimentary rocks hold true in the case of minerals, as well. Further accumulation of experimental data is necessary, however.

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GEOCHEMISTRY

A CONTRIBUTION TO THE PETROCHEMICAL CHARACTERISTICS OF THE GRANITES OF THE NORTHERN PIEDMONT REGION OF THE ALTAI* (AREA BETWEEN THE CHARYSH AND KATUN RIVERS IN THE OB BASIN)

By P. I. LEBEDEV, Corresponding Member of the Academy

Certain granites of the northern piedmont region of the Altai (Belokurikha, region of the Tochilny village, Baurgan Range), as well as of granites from the environs of the town of Novosibirsk, have been studied with the aid of chemical and spectrum analysis. A comparison of the data obtained with the data available on the granite intrusives of the iron-ore regions of the Kuznetsk Alatau, has shown that these granites are characterized by the following features ([1]):

1) absence of traces of Ni and Co, Cr is present in the form of negligible traces, if at all;

2) the presence of Sn and W is clearly pronounced, while no traces of Mo could be detected;

Petrochemical Characteristics of Granites in the Area between the Charysh and Katun Rivers (Ob Basin)

	I	II	III		I	II	III
SiO_2	66.98	73.91	72.40	P_2O_5	0.31	0.11	0.02
Zr	+++	+++	+++	Sn	+++	+	+++
Al_2O_3	14.92	13.61	15.17	Mo	—	—	—
Be		+	++	W	+++	—	+++
Fe_2O_3	1.35	0.99	0.90	Pb	++	+	++
FeO	1.58	0.72	0.29	Cu	—	—	—
TiO_2	0.49	0.18	0.17	CaO	2.93	1.21	0.42
Cr		—	+	BaO	0.10	0.04	0.05
V		+++	+	Sr		++	+++
MnO	0.05	0.32	0.04	$H_2O_{-110°}$	—	0.19	0.34
Ni, Co		—	—	$H_2O_{+110°}$	1.01	0.49	0.21
Mgo	1.71	0.26	0.14				
Na_2O	4.40	3.08	4.56				
K_2O	4.03	5.05	5.36	Total	99.86	99.86	100.27

I, granite of Novosibirsk, V. A. Egorov, analyst; II, granite of the Tochilny village, V. A. Egorov, analyst; III, granite of the Baurgan Range, V. A. Moleva, analyst. + denotes traces of spectral lines; ++, very weak lines; +++, weak lines; —, lines of medium intensity and above.

* For the explanation of the term «petrochemical characteristics», see ([1]).

3) a lower content of Ti, the presence of which is characteristic, *e. g.* of the granitoids of the Tey iron-ore deposit (eastern slope of the Kuznetsk Alatau).

These granites which (unlike the granitoids of Mt. Shoria) are distinguished by the absence of any trace of contamination with derivatives of basic rocks, may, on the basis of their chemical nature, be referred to derivatives of «the pure granite type» and considered as potentially containing rare metals.

These conclusions are in good agreement with the data on the geological and petrographical structure of the northern piedmont regions of the Altai (as *e. g.* the Belokurikha intrusives), where there are no independent manifestations of basic magma.

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BIOCHEMISTRY

THE COURSE OF ACCUMULATION OF ASCORBIC ACID IN THE LEAVES OF DIFFERENT SPECIES OF WILD ROSE

By M. A. ROZANOVA and V. A. VADOVA

(Communicated by A. A. Richter, Member of the Academy, 13. V. 1945)

The course of accumulation of ascorbic acid has largely been studied at the various stages of fruit development, and much less so in other organs, inclusive of the leaves. The study of the accumulation of ascorbic acid in the fruit made it possible to establish that this process does not assume a similar course in all plants, and that the maximum of ascorbic acid accumulation may correspond to different development stages of the fruit.

At the present time it is possible to discriminate three groups of plants, characterized by a different course of accumulation of ascorbic acid in their fruits. Within the first group the maximum of accumulation of ascorbic acid is observed at the stage of unripe fruit; among these should be classed the various species of walnut, red and black currant. The plants of the second group show a maximum of accumulation of ascorbic acid at the stage of ripening, but underripe fruit; among these belongs the pear. The third group comprises such plants as display a maximum of ascorbic acid accumulation at the stage of ripe fruit. To this group are referrable most of the plants which have served for investigation, as for example, wild rose, pepper, tomato, strawberry, raspberry, *Actinidia*, and many others.

As to the process of accumulation of ascorbic acid in the leaves, on the basis of rather scanty literary data on the subject it is possible likewise to differentiate three groups of plants in which the accumulation of ascorbic acid proceeds according to sharply different curves. The first group displays a maximum of accumulation of ascorbic acid at the period of flowering, while at the approach to the stage of fruit-formation the curve makes a sharp bend downward. To this group of plants belong *Fragaria grandiflora* [4]. *Pisum sativum* [3], various species of *Juglans* [6], species of *Gladiolus* [2,5], *Medicago sativa* [2] and number of other plants [7,8]. The plants of the second group display a maximum at the outset of leaf development; towards the period of blooming the curve has a downward run; a single plant only has still been referred to this group, *viz. Medicago sativa* [1]. The plants of the third group show a maximum content of ascorbic acid at the stage of unripe, green fruit. To this group belongs *Fagus silvatica*, although the author did not specify the phenological stage exactly [8].

The course of the accumulation of ascorbic acid in the leaves of wild roses has not been investigated at all. The present authors made it their object to find out whether the amount of ascorbic acid contained in the leaves of wild roses is or is not correlated to that contained in their fruit; we intended furthermore to investigate the course of accumulation of ascorbic acid in the leaves of the wild rose species different as to their vitamin activity.

Accumulation of Ascorbic Acid in Leaves of Different Species of the Genus *Rosa*

Species	Content of ascorbic acid in dry fruit pulp, in mg %	Number of shrubs	Date of analysis	Phenological stage*	Content of ascorbic acid in leaves, in mg %**	Variation in content of ascorbic acid in leaves, in %***
R. cinnamomea L.	7464.4	7	16.VI	Budding	350.56 1009.35	—
			11.VII	Fruiting I	473.85 1338.0	+32.56
			18—24.VIII	Fruiting IV	540.38 1212.51	+20.12
			7.X	Fruiting V	354.25 777.35	−22.96
R. pommifera Herrm.	6964.1	5	20—24.VI	Budding	299.85 796.25	—
			11.VII	Fruiting I	425.7 1080.9	+35.29
			17—18.VIII	Fruiting II—III	450.25 1042.02	+30.86
				Fruiting V	438.50 888.0	+11.52
R. glauca Pourret	6582.7	2	29.VI—8.VII	Flowering	307.1 690.1	—
			19—21.VIII	Fruiting II—III	453.90 1012.65	+46.74
			9.X	Fruiting V	389.20 862.0	+24.91
R. tuschetica Boiss.	5956.9	1	8.VII	End of flowering	392.6 872.5	—
			17.VIII	Fruiting II	488.4 1080.9	+23.88
R. acicularis Lindl.	5510.2	4	10.VII	Flowering, end of flowering	443.48 1019.94	—
			19.VIII	Fruiting III	530.42 1224.95	+20.1
R. Afzeliana Fries.	5486.6	2	10.VII	Flowering, end of flowering	418.25 887.3	—
			24.VIII	Fruiting II	604.0 1356.35	+52.71
R. rugosa Thunb.	5301.1	3	10—22.VII	Flowering and fruiting I	339.17 739.93	—
			19—23.VIII	Flowering and fruiting III	426.03 937.03	+26.65
			7.X	Fruiting IV, V	412.7 889.4	+20.20

Species	Content of ascorbic acid in dry fruit pulp, in mg %	Number of shrubs	Date of analysis	Phenological stage *	Content of ascorbic acid in leaves, in mg % **	Variation in content of ascorbic acid in leaves, in % ***
R. elasmacantha Trautv.	3712.1	2	29.VI	End of flowering	368.5 / 969.9	—
			11.VII	Fruiting II	396.3 / 1042.9	+7.52
			24.VIII	Fruiting IV	408.30 / 944.80	—2.58
			7.X	Fruiting V	412.7 / 906.5	—6.54
R. elliptica Tausch.	3002.0	1	8.VII	Flowering, end of flowering	337.2 / 821.1	—
			21.VII	Fruiting II	485.0 / 1184.4	+44.24
			9.X	Fruiting V	368.8 / 898.4	+ 9.41
R. oxyacantha M. B.	1931.0	2	8.VII	Fruiting I	377.70 / 796.70	—
			21.VIII	Fruiting IV	347.79 / 776.67	— 2.64
			9.X	Fruiting V	266.3 / 570.9	—28.34
R. spinosissima L.	1545.0	3	15—29.VI	Onset of flowering	335.17 / 835.33	—
			12.VII	Fruiting II	385.6 / 930.5	+11.26
			18.VIII	Fruiting IV	419.9 / 1011.7	+21.40
			7.X	Fruiting V	404.1 / 934.0	+11.74
R. micrantha Smith	1486.3	4	27.VI	Onset of flowering	394.4 / 882.3	—
			10.VII	End of flowering	400.67 / 938.80	+ 6.40
			19—24.VIII	Fruiting II	485.42 / 992.90	+12.53
R. multiflora Thunb.	916.4	2	11.VII	End of flowering	351.05 / 1017.4	—
			19.VIII	Fruiting II	375.30 / 1110.9	+ 9.19

* Stages of fruit development: I. young fruit after shedding of petals; II, fruits green, slightly pigmented; III, fruits coloured but unripe (orange in certain species, violet or brown in others); IV, fruits ripe; V, fruits overripe.

** The numerator denotes the content of ascorbic acid in leaves per raw weight; the denominator, per dry weight.

*** The change in the content of ascorbic acid in leaves has been deduced from a comparison with the first phenological stage recorded.

A total of 13 wild rose species were analysed, 7 of them belonging to the active type, containing above 5000 mg per cent of ascorbic acid to the dry fruit pulp: 2 species of medium activity, containing from 3000 to 4000 mg per cent, and 4 slightly active species, containing up to 2000 mg per cent to the dry pulp of the fruit. For some species, as for example *Rosa cinnamomea*, *R. pomifera*. *R. acicularis*, *R. rugosa*, *R. spinosissima*, *R. micrantha*, leaves were taken from 3—7 shrubs (an average sample was taken from shrubs one-year-old and from three-year-old blossoming shoots). For the rest of species samples were taken from 1—2 shrubs. Sampling was done at various phenological stages. The species known to be highest in vitamin (as, for example, *Rosa cinnamomea*) was taken samples of leaves four times in the course of the summer season at various phenological stages, and so was the species lowest in vitamin, *R. spinosissima*, while the other species were examined twice or three times during the summer. The determination of ascorbic acid in fruits and leaves was made by the method of titration of acid extracts with the aid of Tilmans' indicator (0.001 *N* solution of 2,6-dichlorphenolindophenol). Each analysis was made twice, and the mean was adopted as a figure expressing the content of ascorbic acid in the sample. The data obtained are summed up in the table.

From the data tabulated the following conclusions are to be made:

1. We failed to observe any positive correlation between the content of ascorbic acid in leaves and its content in fruits. Thus, for example, *Rosa rugosa* which, according to our analyses, contains 5301 mg per cent of ascorbic acid to the dry weight of the fruit pulp, has been found to contain in the leaves 889 mg per cent to dry weight at the stage of ripe fruit. *R. spinosissima*, whose content in ascorbic acid is 1545 mg per cent, *i. e.* three times less, has been found to contain 1012 mg per cent of the vitamin in its leaves, *i. e.* 1.2 times as much as the former species. *R. glauca*, whose fruits possess an activity of 6582 mg per cent to the dry pulp of the fruit, contains 1012 mg per cent in the leaves at the stage of ripening coloured fruits. *R. multiflora*, characterized by the least degree of activity and which contains 916 mg per cent to the dry weight of the fruit pulp, has been found to contain up to 1110 mg per cent of ascorbic acid in the leaves, *i. e.* nearly as much as the preceding species.

2. The range of specific variation in the content of ascorbic acid in the fruit is appreciably higher than that in the content of the vitamin in the leaves. Thus, in the species investigated by the present authors, the content of ascorbic acid in the fruit varies within a range from 916 to 7464 mg per cent to the dry pulp of the fruit; in the leaves it varies from 571 to 1356 mg per cent to the dry weight, *i. e.* in the former case the maximum index exceeds the minimum 8 times, and in the latter case, 2.3 times.

3. The maximum of accumulation of ascorbic acid in leaves corresponds to the stage of the appearance of young fruits, green or but slightly coloured. After this a decrease in vitamin accumulation sets in, as a rule. An exception to this rule is offered by *Rosa spinosissima*, in which the maximum corresponds to the stage of ripe fruit. According to the mode of accumulation of ascorbic acid, the wild rose should thus be classed among the third group of plants, characterized by a maximum of accumulation of the vitamin in leaves at the stage of ripening fruit, as, for example, *Fagus silvatica* ([8]).

4. The accumulation of ascorbic acid in the leaves of highly active or poorly active species of the genus *Rosa* proceeds in a similar way. The only difference is that the former accumulate the vitamin during the early stage of fruit ripening at a somewhat higher rate than the latter. Thus, for example, in highly active species the difference in the content of ascorbic acid at the stage of budding, or onset of blossoming, fluctuates between 20.1 and 52.71 per cent; in poorly active species, between 12.53 and 21.40 per cent. In such species as occupy an intermediate position no regularity in the variation in the content of ascorbic acid is traceable. Thus, *Rosa elasmacantha* shows an increase by 7.52 per cent, *R. elliptica*, by 44.24 per cent.

5. Highly active species reveal a somewhat more intense reduction in the content of ascorbic acid in their leaves at the stage of ripe fruit than this is observable in poorly active ones. Thus, in *R. cinnamomea* the content of ascorbic acid decreases by 12 per cent within the period from young fruit to ripe fruit, while in the poorly active species *R. oxyacantha* the respective figure is 2.6. In *R. spinosissima* no decrease whatever could be observed.

Towards the stage of overripening of fruit a reduction in the content of ascorbic acid has been recorded both in leaves and fruit of all species investigated.

Thus, highly active species are characterized by their ability more intensely to accumulate the ascorbic acid in their leaves by the time when the vitamin does merely begin to accumulate in the fruits; they are also able to reduce it more intensely towards the stage of ripe fruit, when the amount of ascorbic acid accumulated has attained its maximum in the fruits.

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PÉTROGRAPHIE

MODIFICATIONS PAR ALTÉRATION DANS LA COMPOSITION, LA STRUCTURE ET LES PROPRIÉTÉS DES ARGILES DU CRÉTACÉ INFÉRIEUR

Par M. V. TCHOURINOV

(Présenté par F. P. Savarensky, de l'Académie, le 8. VI. 1945)

Au cours d'études poursuivies dans la ville d'Oulianovsk l'auteur a établi l'existence d'une «zone d'altération» dans les terrains argileux du Crétacé inférieur développés dans la région du point de partage entre la Volga et la Sviaga et sur les pentes de leurs berges.

L'épaisseur de la «zone d'altération» dans la région du point de partage varie de 10 à 15 mètres et diminue sur les pentes jusqu'à 5—1.5 mètres. Comme limite inférieure de la «zone d'altération» on a admis conventionnellement la profondeur, à laquelle l'argile noire ne porte aucune trace d'oxyde de fer, contient de la marcasite et n'a pas d'inclusions de gypse.

Sur les pentes des berges de la Volga la «zone d'altération» est recouverte d'une couche de terrains glissés, dont l'épaisseur atteint 25 mètres au bas de la pente et qui est connue sous le nom de «zone d'accumulation de terrains glissés» (ou «zone d'accumulation des éboulements»). L'étude des éboulements (ou des glissements) a fait voir que tous les mouvements du sol actuels, tant grands que petits, sont entièrement localisés dans la «zone d'accumulation de terrains glissés» et étroitement liés à la «zone d'altération» de la région du partage des eaux et des pentes.

Cette particularité constatée par l'auteur l'a convaincu que l'altération des terrains argileux qui forment la pente joue un rôle important, en préparant, pour ainsi dire, les éboulements (ou glissements). Dans le but de vérifier cette supposition l'auteur se proposa d'établir les modifications de la constitution minéralogique et chimique que subissent les argiles du Crétacé inférieur par altération et les changements de leur structure et de leurs propriétés qui s'en suivent.

Composition minéralogique et structure. Les terrains argileux des quatre étages du Crétacé inférieur (Hautérivien, Barrémien, Aptien et Albien), qui constituent les berges de la Volga dans la région d'Oulianovsk, ont une composition minéralogique à peu près identique. Les principaux minéraux plongés dans la masse argileuse sont: le quartz, le feldspath, la glauconie, et en quantité moindre le mica—la mouskovite et la séricite. Parmi les minerais on trouve constamment de la marcasite en quantité considérable, et parmi les minéraux accessoires c'est l'épidote, le grenat et le zircone, qui prédominent. Les restes de matière organique sont finement dispersés: ils se trouvent aussi en forme de fusains.

Suivant la quantité d'alévrite et de sable fin qu'ils contiennent les terrains argileux du Crétacé inférieur sont caractérisés par une structure péliteuse, alévriteuse et aréno-péliteuse. D'autre part, tous les terrains, indépendamment

de leur structure, sont très compactes et entièrement privés de fissures microscopiques.

Les modifications suivantes s'opèrent à la suite de l'altération: 1) le gypse remplace la marcasite dans les argiles, 2) la structure des terrains argileux devient moins compacte et des fissures microscopiques apparaissent.

Composition chimique élémentaire. En ce qui concerne la composition chimique élémentaire des formations argileuses, non modifiées par l'altération, de même que leur composition minéralogique, les résultats des analyses ont confirmé l'identité de ces formations à tous les stages du Crétacé inférieur. Leur composition chimique moyenne est indiquée dans le tableau 1 (en pour cent).

Tableau 1

Perte à la calcination	SiO_2	Al_2O_3	Fe_2O_3	CaO	MgO	SO_3	K_2O	Na_2O	$\frac{SiO_2}{R_2O_3}$	$\frac{Al_2O_3}{Fe_2O_3}$
De 4	52	15	5	1	1.1	1	2.8	0.6	1.9	2.5
A 9	70	22	8	1.8	1.7	6	3	1.3	2.4	3.5

La comparaison des données du tableau 1 avec la composition chimique élémentaire des formations soumises à l'altération ne révèle pas de modifications importantes. Cela peut être expliqué par la quantité minime des éléments que l'altération fait disparaître. Une analyse brute ne peut pas fixer les modifications survenues, car elle ne nous révèle pas le caractère des associations chimiques sous la forme desquelles les oxydes se trouvent dans les argiles.

Analyse par voie humide. Les résultats de nombreuses analyses par voie humide, faites dans des conditions identiques tant pour l'argile non altérée que pour l'argile ayant subi l'altération, sont indiqués dans le tableau 2.

Tableau 2

Composition des extraits par voie humide de 100 g de terrain dans 500 cm³ d'eau	Terrains argileux du Crétacé inférieur non altérés	Terrains argileux du Crétacé inférieur altérés
Résidu compacte en mg	95—152 *	700—2000
$Ca^{\cdot\cdot}$ mg/équiv	0.2 —1	5—14
$Mg^{\cdot\cdot}$ »	0.13—1	3—11
$Na^{\cdot}$ »	1—7	1—7
$SO_4^{\prime\prime}$ »	0.5 —9.4	9—28
$HCO_3^{\prime}$ »	0.1 —2.4	0.02—0.7
$Cl^{\prime}$ »	0.01—0.2	0.05—0.75
Compositions organiques	5—20	1—7
Concentration des ions d'hydrogène pH	7.2	6.8
Coefficient d'alcalinité $\frac{Na^{\cdot}}{Ca^{\cdot\cdot}+Mg^{\cdot\cdot}}$	1.5—20	0.05—0.7
Coefficient sulfatique $\frac{SO_4^{\prime\prime}}{HCO_3^{\prime}+Cl^{\prime}}$	0.3 —0.8	12—60

* 200—400 pour l'étage Hautérivien.

Comme on peut le voir sur le tableau 2:

a) Lorsque l'altération a lieu, il se produit dans les argiles du Crétacé inférieur des modifications dans le sens de l'accroissement de la quantité des sels solubles, principalement des sulfates de chaux et de magnésium.

b) L'accroissement des sulfates en cas d'altération se produit par oxydation et décomposition des pyrites présents dans les formations argileuses.

c) L'accroissement du Ca^{++} s'explique par la décomposition des concrétions calcaires qui se trouvent souvent dans les argiles du Crétacé inférieur.

d) L'accroissement du Mg^{++} en cas d'altération se produit probablement par suite de la décomposition du mica et de la glauconie.

e) La quantité de composés organiques dans les formations argileuses diminue en cas d'altération par suite d'une oxydation partielle de ces composés. La réaction du milieu ambiant change, à en juger par la teneur en pH, dans le sens de l'oxydation.

f) La teneur des argiles en sodium ne change pas sensiblement en cas d'altération. Quant à son importance relative, comparée à la teneur en métaux alcalino-terreux, elle diminue sensiblement par suite de l'accroissement rapide de ces derniers, ce qui est nettement indiqué par l'importance du coefficient d'alcalinite.

g) L'activité de l'altération devient de moins en moins sensible en profondeur. Le pourcentage des alcalis augmente pour les formations non altérées, tandis que le pourcentage des sulfates diminue.

Composition granulométrique. L'analyse granulométrique des argiles du Crétacé inférieur a été effectuée en présence de l'agent de stabilisation NH_4OH, ce qui a assuré la haute dispersion des matières en suspension. De nombreuses analyses de terrains non altérés appartenant à tous les étages de la série du Crétacé inférieur permettent de faire les déductions suivantes:

a) La série du Crétacé inférieur dans son ensemble est représentée par des terrains uniformes, principalement argileux, caractérisés par la similarité de leur structure granulométrique et la quantité minime des particules de 0.25 à 0.05 mm.

b) On trouve cependant des argiles (argiles lourdes) dans la structure desquelles les particules <0.005 mm prédominent largement, ainsi que des terrains argileux (terrains argileux lourds) dans lesquels la quantité de particules vaseuses augmente considérablement.

c) Parmi les couches (les dépôts) de tous les étages du Crétacé inférieur ce sont celles de la base de l'étage Albien et, à un moindre degré, celles du Barrémien qui sont les plus sablonneuses. La quantité des particules d'argile augmente cependant considérablement dans le Barrémien.

Les modifications que produit dans la structure des argiles l'altération sont illustrées par le tableau 3.

Tableau 3

Particules composantes, mm	Argiles du Crétacé inférieur non altérées, %	Argiles du Crétacé inférieur altérées, %
0.25—0.05	2—3	8—14
0.05—0.005	30—50	60—70
< 0.005	40—60	21—30

Ces modifications sont dues à une augmentation considérable du Ca^{++} dans les argiles altérées. Le Ca^{++} agit sur les particules finement dispersées des argiles, ce qui mène à la formation de plus grandes aggrégations calcaires et à l'apparition d'une structure granulée secondaire dans les argiles.

Humidité naturelle et degré de consistance. Les résultats de nombreuses investigations pour déterminer l'humidité naturelle et la consistance des argiles du Crétacé inférieur altérées et non altérées sont indiqués dans le tableau 4.

Tableau 4

	Argiles du Crétacé inférieur non altérées, %	Argiles du Crétacé inférieur altérées, %
Humidité naturelle	25–29	31—34
Limites de plasticité	29 35	28—31
Limites de fluidité	70—80	50—65

L'examen de ce tableau permet de faire les déductions suivantes:

a) Le processus d'altération dans les argiles du Crétacé inférieur modifie leur humidité naturelle et leur consistance.

b) Dans les gisements d'argiles du Crétacé inférieur l'humidité naturelle des couches qui se trouvent au-dessous des zones d'altération n'est pas relativement grande. Elle n'est ordinairement que de 25 à 29 pour cent, et généralement moindre que l'humidité à la limite la plus basse de la plasticité.

c) La particularité indiquée témoigne que dans les gisements des argiles du Crétacé inférieur ces terrains sont caractérisés par une consistance semi-solide (l'humidité naturelle est inférieure à la plus basse limite de la plasticité).

d) La grande valeur de fluidité des argiles du Crétacé inférieur dans les gisements au-dessous de la zone d'altération permet de constater qu'en cas de modification de leur structure naturelle ces argiles peuvent devenir très plastiques, et gardent cette plasticité même quand leur humectation atteint 80 pour cent.

e) En cas d'altération l'humidité naturelle des argiles du Crétacé inférieur augmente jusqu'à 31—34 pour cent. En règle générale elle dépasse l'humidité minimum nécessaire à leur plasticité, ce qui est démontré par leur passage à l'état plastique, tout différent de la consistance semi-solide des argiles non altérées.

f) La limite de fluidité des argiles du Crétacé inférieur altérées diminue considérablement (de 50 à 65 pour cent) par rapport à la limite de fluidité des couches d'argile qui n'ont pas subi d'altération dans les gisements (de 70 à 80 pour cent). Il se produit simultanément une diminution du pourcentage de plasticité, ce qui permet de constater la moindre étendue des qualités plastiques des argiles altérées et leur passage plus rapide à l'état fluide en cas d'humectation.

g) La limite de plasticité des argiles altérées diminue quelque peu, ce qui permet d'apprécier la grande quantité d'humidité que doivent perdre ces argiles par dessiccation pour passer à l'état semi-solide. Cette quantité est bien plus grande que celle que doivent perdre dans le même cas les argiles non altérées des gisements naturels.

Porosité, densité et cohésion des terrains. Les essais au choc de compressibilité et de résistibilité des terrains, effectués suivant la même méthode sur un grand nombre d'échantillons d'argiles du Crétacé inférieur tant altérées que n'ayant pas subi d'altération, permettent d'établir les particularités suivantes (tableau 5).

Il résulte donc que:

a) Les argiles du Crétacé inférieur qui se trouvent dans les gisements au-dessous de la zone d'altération sont caractérisées par une grande densité et une grande cohésion, ce qui est témoigné par la valeur minime du coefficient de porosité, l'accroissement infime de la densité sous la pression de 15 kg par cm², ainsi que par la grande cohésion et la valeur correspondante de résistance au cisaillement.

b) En cas d'altération, une nouvelle structure apparaît dans les argiles sous l'influence des ions de calcium, la porosité et l'humidité s'accroissent, ce qui, de même que la fissuration, commune aux argiles altérées, diminue sensiblement la cohésion générale des argiles et détruit leur structure.

Tableau 5

Nature des formations	Exposants comparés								Cohésion, kg/cm²
	Coefficient de porosité naturelle	Diminution de volume sous une charge de 15 kg/cm²	Coefficient de porosité après gonflement	Pression naturelle, kg/cm²	Résistance au cisaillement sous une charge, kg/cm²				
					2	3	4	6	
Argiles non altérées de	0.732	0.03	0.800	2.5	1.7	1.8	2.3	2.7	1.1
Argiles non altérées à	0.890	0.06	1.092	5.3	3.3	4.0	4.4	4.8	3.2
Argiles altérées de	1.152	0.10	0.887	0.8	1.0	1.5	1.7	2.3	0.5
Argiles altérées à	1.416	0.49	1.112	1.15	1.2	1.6	1.9	2.6	0.75

La modification de la structure des argiles par altération est démontrée par le fait qu'en cas de gonflement par humidité de l'argile compacte, le coefficient de porosité dans les argiles non altérées est toujours plus grand que le coefficient de porosité initial, tandis que dans les argiles altérées c'est le contraire.

En résumant tout ce qui a été dit ci-dessus, nous pouvons constater que:

a) L'altération des argiles du Crétacé inférieur modifie leur composition minéralogique et chimique, ce qui à son tour cause un changement dans la structure et les propriétés de ces terrains dans le sens de l'accroissement de leur densité et de leur cohésion.

b) On a pu observer à Oulianovsk la localisation des éboulements et des glissements dans la zone des terrains altérés de la partie supérieure de la pente, ainsi que dans la «zone d'accumulation de terrains glissés», composée par des terrains qui ont subi l'altération antérieurement. Ces observations permettent d'envisager l'altération comme une des principales causes des éboulements actuels.

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SOIL SCIENCE

TYPES AND STAGES OF SALINIZATION OF IRRIGATED SOILS AND MEASURES TO BE TAKEN IN ORDER TO CONTROL IT

By V. A. KOVDA

(Communicated by L. I. Prasolov, Member of the Academy, 28. II. 1945)

The salinization of irrigated soils proceeds stagewise. The earliest stage, which takes place over newly irrigated territories, is the salinization of soils along new irrigation channels, caused by important water-losses on percolation, and by a considerable rise of the groundwater level within the zone adjoining the channels. As known by experience based upon reclamation work carried on at Golodnaya steppe and Ferghana, the salinization of soils along new irrigation channels sets in by the second or third year of work, over an area of 300—700 m at either side of the new channel.

The principal measure to be taken in order to prevent soil salinization at this stage is a thorough control of the waters percolating from the channels. The percolation of water both within the irrigation network and over the irrigated fields, where it may be caused by vicious water-supply, goes rapidly to raise the level of groundwater in the oasis. Salinization may gradually invade a considerable portion of the irrigated land, unless necessary measures are taken to improve water-supply under conditions of naturally undrained or poorly drained oases. At this stage of the process a number of minor phases of forms of salinization can be distinguished.

a) Seasonal (summer) salinization of soils takes place in the case of insufficient watering or levelling of the plots, or of excessive evaporation, as well as in the case of a poor development of agricultural crops over the plot. This mode of secondary salinization can readily be controlled, provided rational crop-rotation is introduced; the steps to be taken consist in a decrease of the level of groundwater by an adequate irrigation system, applying rational watering and advanced agrotechnique. To these measures, aside from levelling, belong thorough mechanized tilling, tending to keep the structure of the soil mellow and loose, and an abundant growth of alfalfa included into the crop-rotation. Because of its seasonal occurrence this mode of salinization is sometimes overlooked or undervaluated. The yield of agricultural crops may be considerably increased by preventing and controlling seasonal salinization; besides, the necessary procedures do not require important outlays. Seasonal salinization tends to increase, however, and may become continuous, spotty salinization, unless adequate measures are applied.

b) Continuous spotty salinization develops at fields with an uneven microrelief, because of the horizontal and vertical redistribution of salts in the soil adjoining these plots. This mode of salinization may cause heavy losses to the national economy; it is responsible for the spotty development, or

failure to develop of agricultural crops over about 15—25 per cent of the irrigated area of Golodnaya steppe, Vakhsh, Bukhara, Mugan, *etc.*

Secondary salinization of a spotty type may readily be prevented and controlled with the aid of the following measures: regular levelling of the irrigated territory, decrease of the level of groundwater by improving the exploitation of irrigation system, introduction of rational crop-rotation involving alfalfa, high agrotechnique, in particular, the organization of thorough watering, which must be practised with special care over the solonchak spots.

Quite imperative is also selective autumn—winter washing-watering of the solonchak spots, which is necessary for their desalinization. By overcoming this mode of salinization with the aid of the simplest procedures enumerated above, it is possible to increase the yield of agricultural crops by 15—25 per cent, merely at the expense of the barren spots reclaimed.

c) Continuous salinization associated with groundwaters mineralized up to 10—12 g per litre. This type of salinization sets in in poorly drained parts of irrigated oases (depressions of old river beds, cup-like lowlands, peripheral parts of dry deltas). It also takes place in the case where groundwaters occur near the surface (2.0—2.5 m), at vacant plots within oases.

A distinctive feature of this mode of salinization is the presence of salts in the surface layer of soil, where they attain a concentration of 2—3 per cent, while the subsoil horizons are low in salts (0.3—0.5 per cent), and the water used up by the plants is but moderately mineralized (up to 10—12 g per litre). Continuous salinization, along with a moderate mineralization of the groundwaters, is widespread in the soils of Bukhara and Karakul oases, in the central part of Sokh and Isphara oases, in Ferghana, in Khoresma. With the aid of winter washing, rational watering practised in summer and advanced agrotechnique, soils of this type may well be used under watered crops, without increasing the coefficient of useful land (CUL) kept at 0.2—0.3.

However, the yield of agricultural crops is not stable under these conditions, and the production obtained is both of inferior quality and reduced quantity, for the salinization processes are not stopped altogether. In order to make the land arable all over its area, and to secure stable high yields, it is necessary to attain a stable desalinization of the tilled horizon and the horizon underlying the latter by a thorough washing made possible by the existence of rare deep collectors.

A land affected by this type of salinization thus requires a number of considerably more energetic measures, if it is to be made arable over a vast area; to these measures belong levelling, continuous washing and excavation of rare deep network of drains, whose effect will be to diverge ground and washing waters. The effect of washing should be fixed by cotton-alfalfa crop-rotation, by watering and the introduction of up-to-date agrotechnique, and by rigorously imposed rules of water-supply, tending to control the rise of the level of groundwater.

d) Continuous salinization associated with a high degree of mineralization of groundwater (15—30 g per litre and higher). This mode of salinization is the most malignant and the most difficult to combat.

It is encountered in the less drained regions of irrigated oases, in such depressions of the relief which serve as receptacles and evaporators of the water in excess, as, for example, the Sarysuy depression in Ferghana, the Sardoba and Shuruzak depressions in Golodnaya steppe, the Karalang depression at the Vakhsh.

In this type of solonchaks which are soft, as a rule, the amount of salts contained on the upper layers of the soil attains 5—15 per cent. The subsoil layers are also high in salts (1.5—2.0 per cent). The groundwaters which usually occur at a depth of 1.5—2.0 m are mineralized to a concentration of brine; they are inaccessible to the plants, and produce an intense salinizing influence upon the soil.

In order to make arable the land affected with continuous salinization associated with a high degree of mineralization of groundwater, washing should be

practised in the course of several years; in addition, it is necessary to keep operating a deep and extensive system of drainage channels and collectors.

Repeated washings are likely to ensure desalinization both of the tilled layers of solonchaks and those underlying them.

Ultimately it will also lead to desalinization of the highly mineralized groundwater.

The effect of washing should be fixed during several years (2—3) by watering practised during autumn and winter season and combined with the introduction of transitional crops (cereals, *Leguminosae*), followed by normal crop rotation.

It is hardly advisable to undertake reclamation of this type of solonchaks, unless a system of deep drainage channels and collectors is available.

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GENETICS

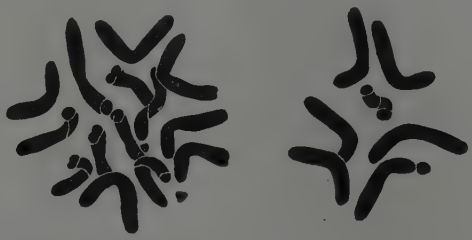
A HAPLOID BARLEY PLANT PRODUCED BY REMOTE HYBRIDIZATION

By V. E. PISSAREV, N. M. VINOGRADOVA and B. A. PODDUBNAYA-ARNOLDI

(Communicated by N. V. Zyzin, Member of the Academy, 7. VII. 1945)

From seeds obtained in 1943 by crossing, following the advice of N. V. Zyzin, Member of the Academy, cultivated barley with *Elymus arenarius* L., there was produced in 1944 a single plant which differed rather strikingly from the mother plant. The latter was *Hordeum vulgare distichum* v. *transcaspicum*: after emasculation its flowers were pollinated with the pollen of *E. arenarius*.

The alleged hybrid plant had attracted our attention, in the first line, by its reduced rate of development and smaller size, and secondly by its pubescence quite unusual for cultivated barley and consisting of sparse and long hairs upon the culm. As regards its development the plant in question was behind its female parent, the



Chromosomes of the mother form of barley and the haploid

rate of growth being about half that of the latter. The characters of the male parent were totally lacking. The situation was such as to suggest cytological investigation of the enigmatic plant which approached in its morphology the maternal form, *Hordeum distichum*.

As a result of a cytological study an atypical chromosome number was found in the plant in question, namely $2n=7$. The cytological analysis has thus revealed the fact that our plant had a haploid chromosome complement and was a haploid mutant. The pollen of *Elymus arenarius* had produced in this case a stimulating effect, causing a parthenogenetic development of the egg-cell, as a result of which an embryo has arisen from the haploid egg-cell without fertilization.

The haploid mutant found by us proved completely sterile and produced no seed.

We have thus obtained through remote hybridization a sterile *Hordeum*-haploid.

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MICROBIOLOGY

THE PART PLAYED BY INDIVIDUAL GROUPS OF LEAVES OF YELLOW LUPINE IN THE GROWTH OF NODULES AND ACCUMULATION OF NITROGEN IN THEM

By M. A. ALI-ZADÉ

(Communicated by A. A. Richter, Member of the Academy, 14. VII. 1945)

The dependence of nitrogen fixation in the nodules of leguminous plants upon the carbohydrate supply of the bacteria has been reported by numerous authors. Fred and Wilson (1) have demonstrated by direct experiments the dependence of nitrogen fixation of the nodule bacteria on the intensity of photosynthesis. Similar conclusions have been experimentally reached by other authors (2). The experimental data available show that the activity of nodule bacteria is entirely dependent on the plant's leaves.

We did not limit our task to the corroboration of the fact that the leaves are the source of supply of assimilable substances to the nodulus and nodule bacteria, but set out to establish what individual groups of leaves play an essential part in these processes.

Pot-culture experiments with yellow lupine (*Lupinus luteus*) planted into soil were arranged with this purpose in view.

The pot-cultures were laid out in 5 kg Wagner containers. After germination 4 plants were left per container. Up to the stage of budding all the plants were kept under identical conditions. At the beginning of bud formation 28 containers with plants showing a uniform development were selected, and their leaves removed according to the following scheme (Table 1).

Four parallel experiments were thus arranged. On the day of the initiation of the experiment (the beginning of bud formation) in 4 of the containers the roots of the plants were washed and the nodules subjected to analysis. The removal of the leaves on the test-plants was done in the following way: the leaves on the main stalk were divided into three groups according to the number of nodes—the lower, middle and upper. Each group consisted of 8 leaves. According to the scheme of the experiment the leaves were removed by groups. Together with the leaves we removed also the axillary buds. The removal of the leaves was started at the time of bud-formation and continued up to the end of the experiment. The buds were regularly removed from the plants, the controls included.

20 days after the initiation of the test the root-system of the plants in all the sets of the tests was washed, the nodules were separated from it and their content in total and protein nitrogen was determined. Total nitrogen was determined by the micro-Kjeldahl method; the nitrogen of proteins, by that of Bernstein; the non-proteinic nitrogen, from the difference between the first two determinations.

Table 1 shows the air-dry weight of the nodules per one plant.

Among the sets in which only one individual group of leaves was removed, the removal of the upper group of leaves had a particularly strong effect; it brought

Table 1

Experimental sets	Air-dry weight of nodules (g)	Difference from the weight of nodules in the control plants	
		absolute (g)	per cent
Plants with intact leaves (control)	0.630	—	100
Lower leaves removed	0.611	—0.019	97
Middle » »	0.500	—0.120	80.9
Upper » »	0.457	—0.173	72.5
Lower anb middle leaves removed	0.405	—0.225	64.3
Lower and upper » »	0.325	—0.305	51.6
Middle and upper » »	0.268	—0.342	39.1
At the moment of bud-formation, *i. e.* when the removal of the leaves was started	0.237	—0.393	35.6

about a 27.5 per cent decrease in the dry weight of nodules. Among those in which we removed two groups of leaves, the strongest effect was produced by the elimination of the middle and upper leaves. In this case the dry weight of the nodules was reduced by 60.9 per cent as compared with the controls. In the course of the 20 days from the beginning to the end of the experiment, the nodules on the plants from which the middle and upper leaves had been removed showed scarcely any growth. On the other hand, the removal of the lower leaves alone produced scarcely any influence upon the growth of nodules and accumulation of mass.

The results of the total and protein nitrogen content determinations in the nodules are highly interesting (Table 2).

Table 2

Experimental sets	Total nitrogen	Protein nitrogen	Non-proteinic nitrogen	Protein nitrogen in % of total nitrogen
Plants with intact leaves (controls)	5.55	4.32	1.23	79.8
Lower leaves removed	5.61	4.06	1.35	72.2
Middle » »	5.52	4.01	1.51	72.6
Upper » »	4.79	3.87	0.92	80.8
Lower and middle leaves removed	—	—	—	—
Lower and upper » »	4.36	3.78	0.58	86.7
Middle and upper » «	4.09	3.85	0.24	94.1
At the moment of bud-formation, *i. e.* when the elimination of the leaves was started	5.61	4.82	0.79	85.9

The removal of the lower and middle leaf groups individually did not affect the per cent content of total nitrogen of the nodules, but it reduced their per cent content in protein nitrogen. The elimination of the upper leaves decreased greatly the per cent content of both total and protein nitrogen, the decrease in the amount of non-proteinic nitrogen being particularly great, this leading to an increase in the ratio of protein nitrogen to the total. A similar effect was produced by a simultaneous removal of the upper and middle leaves. In this case the amount of non-proteinic nitrogen was greatly reduced, and accordingly so was also that of total nitrogen. The protein nitrogen was decreased to a relatively lesser degree. Therefore the ratio of protein nitrogen to the total has a higher value, being the maximum of those for all the sets of the experiment.

By means of suitable calculations we obtain from Tables 1 and 2 the figures of absolute accumulation of both total and protein nitrogen in the nodules (in mg per container). These data are presented in Table 3.

Table 3

Experimental sets	Total nitrogen	Protein nitrogen	Non-proteinic nitrogen	Ratio of protein nitrogen to non-proteinic one
Plants with intact leaves (controls) . .	34.96	27.21	7.75	3.5:1
Lower leaves removed	34.27	24.80	9.47	2:6:1
Middle » »	27.60	20.05	7.55	2:6:1
Upper » »	21.84	17.68	4.16	4:2:1
Lower and middle leaves removed . . .	—	—	—	—
Lower and upper » » . . .	14.17	12.28	2.89	4.2:1
Middle and upper » » . . .	10.96	10.31	0.65	15.8:1
At the moment of bud-formation, *i. e.* when the removal of the leaves was started	13.29	11.42	1.77	6.4:1

In every case, except the total nitrogen in the second set, the removal of leaves leads to a decrease in the amount of both total and protein nitrogen in the nodules. A considerable decrease in the total amount of both protein and non-proteinic nitrogen is observed after the removal of the upper leaves. The removal of the middle group of leaves has also affected the content of the nodules in both forms of nitrogen. The removal of the lower leaves, on the other hand, decreased the amount of protein nitrogen only to the extent of 2.41 mg per container, the amount of non-proteinic nitrogen rising slightly at the same time.

Among sets in which two groups of leaves were removed simultaneously, an abrupt decrease of nitrogen in the nodules was observed in those plants from which the middle and upper leaves had been removed. In this case strongly reduced is the content of non-proteinic nitrogen. The ratio of protein nitrogen to the non-proteinic one in this case is 15.8 : 1.

The above results show that a strong delay in the development of the nodules and a sharp decrease in their nitrogen content is observed when the upper leaves of the plant are removed. However, it would not be correct to draw from this the conclusion that the upper leaves play the essential part in the carbohydrate nutrition of the nodules. The removal of the upper leaves situated in the younger zone of the plant induces an active growth of fresh leaves and shoots, which in our tests were immediately removed. When the middle leaves are removed, the growth of fresh leaves and shoots, though it takes place too, is relatively weak. In the case of removal of the lower leaves, no new growths whatever were observed. In this case nothing was consumed by the plant for the formation of fresh leaves and shoots. In that of the removal of the upper leaves where at the site of the leaves removed there arose an intense growth of fresh leaves and shoots, a great amount of nutrient substances was used up. This was responsible for the change in the normal carbohydrate nutrition and nitrogen metabolism of the nodules.

Attention should be paid to the circumstance that simultaneously with the relatively low level of protein nitrogen in the nodules, the ratio of protein nitrogen to total nitrogen is considerably higher in the plants with their upper leaves removed than in the other sets (Table 2). This high percentage of protein nitrogen in the total nitrogen content of the nodules is not due to its high content, but to the decrease in the content in non-proteinic nitrogen (Table 2). It seems probable that a large amount of non-proteinic nitrogen has migrated from the nodules to the above-ground organs of the plant.

A similar condition is observed in the plants of those sets in which two groups of leaves were removed. When the middle and upper leaves were removed, the content of non-proteinic nitrogen in the nodules sank to an insignificant value, and accordingly, the percentage of protein nitrogen rose to 94.2. The regularity deduced on the ground of the data presented in Tables 1 and 2 is corroborated by those of

Table 3 concerning the total nitrogen content. The high ratio of protein nitrogen content to that of non-proteinic nitrogen observed after the removal of the upper leaves, together with a very low content in non-proteinic nitrogen, speaks in favour of an increased off-flow of the latter from the nodules to the above-ground organs of the plant.

A similar phenomenon of the migration of nutrient substances under the influence of the removal of the upper leaves in tobacco plants has been experimentally observed by Smirnov [3]. After the upper leaves had been removed, the content of both nitrogenous compounds and carbohydrates in the lower leaves decreased, and a migration of these substances to the zone of growth was observed.

The results of our experiments permit to draw the conclusion that after a corresponding surgical interference with the life-activity of the plant, the nitrogenous compounds of the nodules may migrate intensely to the above-ground organs of the leguminous plant.

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PLANT PHYSIOLOGY

SALT RESISTANCE OF ASPEN UNDER NATURAL CONDITIONS

By I. A. KRUPENIKOV

(Communicated by A. A. Richter, Member of the Academy, 6. VII. 1945)

Many representatives of the genus *Populus* are distinguished by their high resistance to readily soluble salts present in the soil in unusually high quantity. The following species proved to be especially resistant: the white poplar(1), the Canadian poplar (2), the Bachofen poplar and the black poplar (3).

The salt-resistance of aspen (*Populus tremula* L.) has been scarcely studied if at all although this very representative of the genus *Populus* occupies a very vast area and is distinguished by a very broad ecological amplitude. The economic importance of aspen is generally known.

During several years (1936—1942) we have observed many instances when aspen was growing on saline soils in the region of insular woods of North-western Kazakhstan (pine woods Aman-Karagai, Naurzum-Karagai, Kazan-bas; birch-aspen woods Sypsyn-Agash and Bet-Agash). In these woods a strong progressive salinification of the upper horizon of the soil is taking place, which causes an adaptation of the tree-shrub plants to the saline substratum foreign to them. In the case of the birch, the process resulted in a formation of the endemic «saline» species, *Betula kirghisorum* Saw.-Ryczg. which occurs in the extreme southern woods (4, 5) only. Along with pine (7), hedge rose (8), with the red hawthorn, the aspen has formed a «specific» saline ecotype which differs from the common aspen in a number of peculiar characteristics.

Within the region in question aspen occurs on most varied soils, on the primitive sandy-soils, humus-wood soils, on the primitive, often stony soils formed upon the deluvium of the coloured miocene clays and, lastly, on the meadow-solonchak and meadow-saline soils within insular woods and along their outskirts. Soils of the latter type always contain great amounts of lime, sometimes of gypsum, and are more or less rich in readily soluble salts, *viz.* chlorides, sulphates and sometimes, sodium carbonate. The reaction of soils is a clearly alkaline one (Table 1).

The alkaline reaction of the soils (pH = 8.0) and high calcium carbonate content create an unfavourable ecological situation for the development of woody plants. However, under the conditions of insular woods of our region, trees and shrubs became adapted to alkalinity and to high carbonate content of the substratum, and even an exceedingly high $CaCO_3$ content (of the order of 30—45 per cent) causes no chlorosis of the leaves; on the contrary, the leaves are a more vivid green than those of the trees growing in other habitats; this is especially striking toward the end of summer. This was proved by our numerous investigations of the soil from underneath of aspen.

Meadow-solonchak and meadow-saline soils of the region in question are clayey and underlain by sands. These soils are confined to large flat depressions between

Table 1

$CaCO_3$ Content in per cent to Absolutely Dry Weight, and pH of the Soils from Underneath of Aspen

Section, No.	Characteristic of plot and soils	Depth of sampling	$CaCO_3$	pH colorimetrically
1001	Peripheral part of saline meadow in the region of the north-eastern outskirts of Naurzum woods	0—10	22.11	8.0
		20—30	15.46	8.0
		40—50	9.11	7.8
	Meadow-solonchak soil	90—100	6.63	7.9
		130—140	2.17	8.0
1002	Saline meadow at the edge of Sokand-sor in the region of the south-eastern outskirts of the Naurzum woods	0—10	28.42	8.0
		20—30	22.18	8.0
		30—40	19.46	8.0
		60—70	15.74	7.9
	Meadow saline soil	90—100	9.46	8.0
		140—150	3.15	8.0
1009	Saline meadow along the edge of the solonchak-sor at the central part of the Sypsyn-Agash woods	0—10	32.17	8.0
		10—20	45.49	8.0
		20—30	25.86	8.0
		50—60	20.15	8.0
		80—90	12.41	8.0
	Meadow saline soil	90—100	4.15	8.0

sandy hillocks which are characterized by a rather superficial (1.2—2.0 m) occurrence of stagnant underground water rather rich in mineral salts. In summer when intense evaporation takes place there occurs a steady upward migration of the underground water toward the surface. Evaporation results in an increase in the readily soluble salts in the soil; the moisture content of the upper horizons of the soil becomes reduced toward the close of summer.

As a result of the two processes the concentration of salts in the soil solution steadily increases and attains its maximum toward the end of the growing period. The autumnal precipitations and especially the impregnation of the soil with water immediately after snow thawing down to the underground water, result in a secondary washing of the most soluble salts into the underground water. The soil is thus characterized by a «pulsating» salt-regime which has a more or less stable annual rhythm. We have shown elsewhere ([5]) that in the meadow-saline soils from underneath the Kirghizian birch the absolute Cl′ content increased from June to September 1940, in some instances, more than ten times, while the conventional Cl′ concentration in the soil solution increased more than twenty times. In connexion with these findings all the determinations of the readily soluble salts in the soils from underneath the aspen were done in autumn (September, 1941, see Table 2).

The figures presented in Table 2 demonstrate a very high salt-resistance of the aspen studied. Inasmuch as no such instances were recorded in literature and, on the other hand, there is evidence pointing to a very low salt-resistance of aspen ([1]), we are inclined to regard the species here described as a specialized salt-resistant ecotype which is a kind of progressive endemic of the undrained region of the insular woods of the Kustanai region.

The specific character of this edaphic ecotype of aspen consists in a number of peculiarities of habitus, and of ecological and biochemical characters. The «saline» aspen is dwarfed as compared with the common form, its height as a rule not exceeding 5—7 m, while on sandy forest soils of the same region there occur individuals 15 m of height and 35 cm in diameter. The leaves of the «saline» aspen are more succulent, of a deeper green and somewhat thicker than those of the common aspen.

Table 2

Humidity and Chemical Composition of Water Extract from underneath Aspen (for the characteristic of plots consult Table 1)*

Section, No.	State of aspen growth	Depth of sampling	Moisture, %	Cl'		SO_4''		Date
				%	conventional concentration, g/l	%	conventional concentration, g/l	
1001	Very good condition, 8—10 years old, average height 3—5 m, green leaves, abundant undergrowth, 1—3 years old	0—10	18.3	0.260	14.2	0.152	8.3	18.IX
		20—30	13.1	0.185	14.1	0.136	10.4	
		40—50	8.5	0.052	6.0	0.102	12.0	
		90—100	4.2	0.009	2.1	0.011	2.5	
1002	Satisfactory condition, 8—12 years old, average height 3—4 m, green leaves, scarce undergrowth	0—10	21.6	0.357	16.5	0.543	25.1	18.IX
		30—40	17.5	0.311	17.7	0.417	23.8	
		60—70	9.8	0.095	9.8	0.217	22.1	
		90—100	5.8	0.102	17.6	0.087	15.0	
1009	Satisfactory condition, about 10 years old, average height 3—4 m, green leaves	0—10	23.7	0.396	16.7	0.759	32.0	25.IX
		20—30	17.2	0.185	10.7	0.256	14.8	
		80—90	9.1	0.202	22.2	0.311	34.1	

* The conventional concentration of the anions in g/l has been computed after the formula ([2,9])

$$\frac{\%\text{ of anion in dry soil}}{\%\text{ of moisture in dry soil}} \times 1000$$

It is of interest to note that similar changes of habitus are observed in cotton plants cultivated on saline soil (dwarfness, increased general thickness of the leaves, *etc.*) ([10]). All these alterations are probably the result of the adaptation of the plant to saline surroundings.

The biochemical characteristics of the «saline» aspen consist in a high content of mineral materials in the leaves and in a very considerable accumulation of water-soluble stuffs in them. The latter is an especially important adaptation of the plant to salinization, for it promotes a greater osmotic pressure of the cell sap.

Table 3

Total Ash Content (per cent of dry weight of the plant) and Chemical Composition of the Ash (per cent of the weight of ash) in the Leaves of Different Aspens from the Naurzum Woods (September, 18)

Ecotype of aspen	Total ash content	In per cent to weight of ash			
		CaO	K_2O	P_2O_5	SO_3
Common aspen from sandy habitats . .	4.96	38.16	6.82	0.34	1.47
«Saline» aspen	7.12	47.42	5.49	0.51	3.05

The leaves of the «saline» aspen have an ash content one and a half times higher than that of the common aspen. In the ash of the «saline» aspen are accumulated greater quantities of CaO, P_2O_5 and especially of SO_3, which is present in a quantity more than twice exceeding that of the common aspen. It should be noted that the representatives of the genus *Populus* are generally distinguished by their capacity to accumulate great quantities of calcium ([11]).

Table 4

Results of Chemical Analysis of Water Extract of Leaves of Different Aspens, in per cent to the Dry Weight of the Plant (the analyses were made after the method described by M. M. Shchukevich [12], September, 18)

Ecotype of aspen	Solid matter	Water-soluble organic substances	Alkalinity in HCO_3'	Cl'	SO_4''	Ca''	Mg''
Common aspen from sandy habitats	9.91	4.12	0.97	0.12	0.22	0.11	0.12
«Saline» aspen	13.46	7.23	2.09	0.44	0.51	0.24	0.27

With regard to the amount and the character of water-soluble substances in the leaves of «saline» aspen (Table 4) there are also certain biochemical characteristics which sharply distinguish it from the common aspen and remind of such a specialized plant of salinified soils as *Betula kirghisorum* [6]. The principal characteristics are the immense total content of water-soluble materials (13.46 per cent), a rather high alkalinity index and a very great amount of water-soluble organic matter (7.23 per cent) with a moderate mineral matter content, although the relative increase in Cl' in the leaves of the saline ecotype is very significant as compared with the common aspen. One may say in general that the high osmotic pressure of the cell sap of the form in question has been attained mainly at the expense of the accumulation of water-soluble organic materials and to a much lesser degree, at the expense of readily-soluble mineral salts.

The «saline» aspen, like the Kirghizian birch, thus exhibits some biochemical affinity to the so-called organic-mineral plants growing on saline soils.

The saline ecotype of the aspen should widely be used for introduction on salinized soils of Kazakhstan and of other similar regions. The economic importance of the ecotype of aspen here considered is naturally a limited one, but it may have an ornamental application and serve as feed for game (young bark for hares; leaves, buds and catkins, the principal feed of black game and ptarmigans) and in apiculture (pollen and glue from the young leaves).

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PHYSIOLOGY

SYSTEM HISTAMINE—HISTIDINEDECARBOXYLASE—HISTAMINASE IN THE NERVE TISSUE OF HOMOIOTHERMAL ANIMALS SUBJECTED TO DIFFERENT CONDITIONS

By CH. S. KOSHTOJANZ, Corresponding Member of the Academy, D. E. RYVKINA and R. L. MITROPOLITANSKAYA

The suggestion has recently been advanced that along with cholinergic and adrenergic nerves there existed also histaminergic nerves; to this group were referred the nerves responsible for antidromal vase-dilatation (1). Rosenthal (2) has also cited many instances, pointing to that histamine is also a chemical mediator of pain. All these data and many others make its seem expedient to advance the problem as to the rôle played by the system histamine—histidinedecarboxylase—histaminase in the processes bringing about excitation in sense nerves and in nerve cells associated with them. The treatment of this problem is intimately linked with the problem bearing upon the enzymic chemical characteristic of the nervous process which takes places at the various points of the nervous system, being at the same time closely interrelated with a number of urgent problems in the domain of pathology.

The present note contains comparative data on the content of histamine, as well as on the activity of histidinedecarboxylase and histaminase in sense and motor nerves of various animals.

Material and Method. As experimental animals use was made of rats, rabbits, guinea-pigs, cats and dogs. The animals were sacrificed either by the routine method, *viz.* a blow on the head, or else by means of sulphur ether. The results of analyses proved quite different according to the experimental condition adopted. Their comparison, which is given below, went clearly to bear out the peculiar nature of the changes in the system histamine—histidinedecarboxylase—histaminase which take place in the nerve tissue under the influence of the trauma.

Under investigation were the posterior and anterior roots, interspinal ganglia, sciatic nerves and subcutaneous membrane.

The content of histamine was determined according to the method introduced by Barsoum and Gaddum, and recalculated in γ per 1 g of tissue.

Results. Histamine. As pointed out by Kwiatkowski (1), the nerves of different animal species do not contain the same amount of histamine; thus, it shows a maximum in rats and a minimum in rabbits. The results of our analyses are summed up in Table 1.

As may be seen from this table, in all animals sacrificed by means of sulphur ether, histamine is found to be present in the anterior and posterior roots, in ganglia, mixed nerves, and subcutaneous membrane. The posterior roots are found to contain the largest amount of histamine. The high histaminic effect revealed in the posterior roots may possibly be associated with the chemical nature of the sense nerves and with the antidromal action described by Kwiatkowski. At

Table 1

Histamine Content in Nerve Tissue of Animals (in γ per 1 g of tissue)

	Posterior roots	Anterior roots	Interspinal ganglia	Sciatic nerve	Subcutaneous membrane
			Rats		
Sulphur ether	150	74	60	27	30
	170	106	60	23	16
Mean	160	90	60	25	23
Blow on the head	0	0	6	5	7
			Dogs		
Sulphur ether	—	—	30	—	7
	120	75	60	13	—
	100	—	40	26	—
	70	20	38	28	16
	40	33	—	24	12
	—	—	58	25	—
	50	25	—	40	—
	60	70	—	15	—
Mean	73	45	45	24	12
Blow on the head	14	18	—	26	—
	0	0	—	40	—
	0	6	0	0	0
	0	20	0	14	18
	0	60	0	12	—
			Cats		
Sulphur ether	40	—	18	4	6
	30	31	12	5	12
	45	30	17	5	13
	—	32	32	6	13
	30	20	25	14	—
Mean	36	28	21	7	11
Blow on the head	0	16	0	7	10
	0	6	2	7	10
	0	11	18	3	6

this juncture of interest is also the presence of a rather large quantity of histamine in the subcutaneous membrane, a tissue which is supplied with sense nerves and which in animals shows a very rapid reaction for skin traumas, in the form of hyperaemia and oedema.

A blow on the head resulted in all our experiments in a sharp decrease of histamine content or in its total disappearance; it should be added that in all animals it vanished first of all in the posterior roots. Thus, our experiments went to demonstrate that in the case of trauma the tissues of all nerves become poor in histamine, the «effect of disappearance» taking place. And in the first place this disappearance takes place in the sense elements of the nervous system.

Histidinedecarboxylase. In sense, motor and mixed nerves we revealed the presence of active histidinedecarboxylase.

Method. I—experiment. Into Warburg's flasks is placed a weighep portion of tissue cut with scissors, 50—100 mg + 30 mg of histidine (histidinedichloride) + *N*/5 NaOH up to neutralization + 2 ml of Tirode's solution + 1.5 ml *N*/15 sodium biphosphate.

II—control. A similar weighed portion + 2 ml of Tirode's solution + 1.5 ml of sodium biphosphate was saturated during 10 minutes with nitrogen passed through an red hot copper and kept during 3 hours in a Warburg apparatus at 37.8° C. After this the content of the flask was filtered off and tested on a guineapig atropinized intestine. Calculation was made after the increase of histamine content in the experiment as compared to control, in γ per 1 g of tissue. The results of this experimental set are summed up in Table 2.

Table 2

Activity of Histidinedecarboxylase in the Nerve Tissue of Different Animals

	Posterior roots	Anterior roots	Interspinal ganglia	Sciatic nerve
Dogs				
Sulphur ether	15	11	20	0
	18	11	20	0
	28	28	20	0
Mean	20	17	20	0
Blow on the head	25	8	31	14
	30	50	50	13
	66	40	73	20
Mean	40	33	51	15
Cats				
Sulphur ether	25	38	15	17
	35	20	15	18
	28	28	14	14
Mean	29	29	15	16
Blow on the head	90	90	130	—
	100	—	—	—
	55	50	25	50
	50	10	18	25
Mean	74	50	58	37

From the data of Table 2 it may clearly be seen that when the animals are killed with sulphur ether, no pronounced difference can be observed between the activity of histidinedecarboxylase in the posterior or in the anterior roots. In the case the animals were traumatized, the activity of histidinedecarboxylase is found markedly to increase in all tissues, the highest figures corresponding to the sense elements of the nervous system. The very fact of a physiological activation

of the histidinedecarboxylase in an organism subjected to traumatization is of great interest. The mechanism underlying this phenomenon has to be deciphered.

We thus see that in the nervous system of an animal which has been dealt a trauma, favourable conditions are created for the production of increased quantities of histamine, while the latter is found to disappear from its tissues. The question arises, what is then happening to the histamine? At this juncture of special interest are data bearing on the histaminase.

Histaminase. Method. I—experiment. 50—100 mg of tissue ground with quartz sand + 2.0 ml of phosphate buffer with a pH = 7.6 + 5 γ of histamine + + thymol.

II—blind experiment. A similar weighed portion + 2.0 ml of buffer + thymol.

III—control. 5 γ of histamine + 2 ml of phosphate buffer solution + thymol are kept for 24 hours in a thermostat at a temperature of 37.6° C. After this the liquid is filtered off and tested on a guinea-pig atropinized intestine. Calculation was made according to the quantity of histamine decomposed per 1 g of tissue.

The results of this set of our experiments are illustrated by Table 3.

Table 3

Activity of Histaminase in the Nerve Tissue of Different Animals

	Posterior roots	Anterior roots	Interspinal ganglia	Sciatic nerve
		Dogs		
Sulphur ether	50	70	80	0
	70	70	70	0
	30	47	40	20
Mean	50	62	63	—
Blow on the head	16	11	0	10
	0	0	0	0
		Cats		
Sulphur ether	20	35	30	0
	20	15	25	0
	35	25	0	0
	40	70	20	25
Mean	29	36	19	—
Blow on the head	0	0	0	0
	0	13	0	20

From Table 3 it follows that the tissues of the majority of nerves contain histaminase. In the sciatic nerves the activity of histaminase is either low, or labile.

Our data should be taken to mean that in the case of trauma the histaminase is inactivated in the tissue of all nerves.

Similar results have also been obtained when studying the breakdown of the histamine contained in the tissue (Table 4).

Table 4

Intensity of Breakdown of the Histamine of the Nerve Tissue in the Case of Trauma or Ether Narcosis in Dogs (in γ per 1 g of tissue)

	Postertor roots	Anterior roots	Interspinal ganglia	Sciatic nerve	Subcutaneous membrane
	Initial content of histamine (mean)				
Ether narcosis	70	45	41	24	12
	After the tissue pulp was kept 3 hours in a thermostat, in a phosphate buffer solution with pH=7.6				
	30	35	20	10	5
	5	5	5	3	—
	15	—	5	—	4
	38	6	0	6	
	10	25	—	16	12
Mean	19	15	7	9	9
Breakdown in %	70	70	90	60	25
	Original content of histamine				
Blow on the head	14	18	—	26	—
	After a 3-hour stay in the thermostat				
	14	18	—	26	—

The data of Table 4 make it evident that in experiments involving the use of sulphur ether the histaminase of the tissues shows a higher activity, while in the case of trauma the decomposition of the histamine contained in the tissue itself is stopped almost altogether, *i. e.* inactivation of histaminase takes place.

Thus, if an animal is traumatized, in its nerve tissue conditions are created which are favourable for the production of an increased amount of histamine (activation of histidinedecarboxylase), and for its preservation from enzymic decomposition (breakdown), *viz.* inhibition of histaminase. At the same time, the histamine disappears from the tissue («disappearance effect»). It is evidently released from the compounds it makes part of in the protoplasm of the nerve tissue and thus becomes able to contribute to the generel effect upon the organism, which is characteristic of the state of trauma. The alterations described above take place both in sense and in motor nerves.

If we are to compare the systems histamine—histidinedecarboxylase—histaminase in the sense nerves, on the one hand, and in the motor nerves, on the other, we fail to reveal any specificity in the strict sense of the word; in the sense nerves the content of histamine is higher, however, and the effect of disappearance in them sets in sooner and is more pronounced than in the motor nerves.

Neither were we able to bear out in our experiments any specific difference between the enzymic activity. This problem cannot be solved unless the effect of accurately controlled stimulation of sense nerves upon the activity of the enzymes is studied in detail. Along with the analysis of the rôle of the system histamine—

histidinedecarboxylase—histaminase in the nervous activity of various animals, viewed in the aspect of the evolution of the chemical basis of the nervous system, this study will constitute the subject of our further research.

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ZOOLOGIE

UNE NOUVELLE FORME DE CAMPAGNOLS *MICROTUS GREGALIS* PALL. (*MAMMALIA, MURIDAE*)

Par V. G. HEPTNER

(Présenté par I. I. Schmalhausen, de l'Académie, le 25. VI. 1945)

Le musée zoologique de l'Université de Moscou a reçu de E. V. Potekhina trois exemplaires de campagnols *Microtus gregalis* trouvés par E. V. Potekhina et par N. P. Kanaréikina en 1939 dans les environs de la ville Aralskoïé Morié, située sur la rive septentrionale de la mer d'Aral, à l'extrémité nord-est de la baie Perovski.

Cette trouvaille présente un très grand intérêt, car elle a été faite loin de l'aire d'habitat de l'espèce connue jusqu'à présent (limitée au nord par le Kazakhstan du Nord, à l'est—par le Tian-Chan de l'Ouest).

Il est intéressant, d'autre part, de rencontrer l'espèce dans les plaines du Turkestan, car, en général, le *M. gregalis* peut être considéré dans ce pays comme un animal de montagne. On ne les a rencontrés jusqu'ici en plaine que dans les régions d'aval du fleuve Ili, en Semiretchié.

Un grand intérêt présente non seulement le fait que cette espèce ait été rencontrée dans une zone désertique, mais également les conditions biotopiques de son existence. D'après l'information du collecteur, les animaux ont été capturés à une distance de 500—600 mètres du bord de la mer, dans une dépression de terrain entre des monticules de sable d'une hauteur de 1—2 mètres. Ces monticules sont constitués de sable à gros grain, très faiblement recouvert d'herbe et remué par le vent. On y rencontre parmi des herbes peu variées la laîche, des touffes séparées d'iris, des oignons sauvages, de rares tulipes et quelques autres espèces. Les terriers des animaux étaient faits dans les racines des plantes.

M. gregalis appartient aux espèces présentant une grande variabilité écologique. Le campagnol se rencontre dans les steppes partiellement boisées, dans la toundra et dans les montagnes. Cependant jusqu'ici il n'a pas été rencontré dans des régions désertiques. Il est intéressant qu'il cohabite dans ces endroits avec des habitants psamophiles des déserts de l'Asie Centrale, comme la gerbille *Pallasiomys meridianus* et les gerboises.

Les voies qu'a suivies l'espèce pour atteindre la mer d'Aral en venant du nord et de l'est, ne sont pas encore éclaircies. Il se peut que le campagnol a suivi pour atteindre la mer d'Aral la vallée de Syr-Daria ou ses affluents, car la zone montagneuse de l'aire d'habitat de l'espèce s'étend sur une partie du bassin de la rivière précitée.

Les campagnols du bord de la mer d'Aral doivent être séparés et peuvent se caractériser de la manière suivante:

Microtus (*Stenocranius*) *gregalis*, *zachvathini* sbsp. nova

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Locus classicus: rivage nord-est de la mer d'Aral (baie de Perovski) auprès de la ville Aralskoïé Morié.

Diagnose: *M. gr. zachvatkini* se caractérise par de très grandes dimensions, ce qui la rapproche du *M. gr. major* Ogn. La fourrure présente un reflet jaune-roux intense.

Dimensions du type: longueur condylobasale du crâne 30.4 mm; distance entre les arches zygomatiques 16.2 mm; distance interorbitale 3.0 mm; longueur alvéolaire de la rangée dentaire supérieure 7.2 mm.

Cette sous-espèce est dédiée au professeur A. A. Zakhvatkine.

La forme décrite est remarquable surtout par ses grandes dimensions. Le plus grand exemplaire d'une série considérable de *M. gr. major* présente une distance condylobasale de 30.2 mm. Mais en général les exemplaires adultes de cette forme présentent une distance condylobasale de 28—29.5 mm. Ainsi donc la forme nouvelle est même peut-être encore plus grande que *M. gr. major*.

Les races du campagnol *gregalis* que l'on rencontre en U. R. S. S. se laissent classer d'après la teinte de la fourrure sur leur dos et leurs flancs en deux groupes. Le premier comprend les exemplaires présentant une fourrure grise ou gris foncé, couleur souris. Les parties distales jaunes des poils sont très peu développées et ne changent pas la coloration générale de l'animal. Les variations de la teinte suivant les différentes races sont marquées. Mais on ne peut noter qu'une décoloration plus ou moins avancée de la même teinte grise. Le second groupe est caractérisé par un développement considérable des parties distales jaunes-rousses des poils. Leur développement est tellement considérable que la coloration générale de l'animal en devient jaune-rousse, et le gris est recouvert et disparaît.

Le premier groupe comprend tous les sous-espèces que l'on rencontre en U.R.S.S, sauf *major* et *zachvatkini* qui constituent le second groupe*.

Il faut noter que dans ce groupe, et même assez étroit, on rencontre des races qui sont à l'antipode l'un de l'autre comme au point de vue écologique, de même au point de vue géographique.

Tels sont: la race la plus septentrionale et l'une des races les plus méridionales, l'habitant des toundras et l'habitant du désert. Les différences entre eux sont peu marquées et consistent en ce que la race *major* est en général un peu plus foncée et de teinte moins vive, et ceci se rapporte non seulement à la teinte générale de la sous-espèce, mais également au pourcentage assez élevé des individus foncés.

C'est ainsi que *M. gr. zachvatkini* est pareil au *M. gr. major* par ses deux caractères distinctifs principaux.

Si la dimension du *M. gr. major* répond à la loi de Bergmann, le même caractère distinctif du *M. gr. zachvatkini* contredit cette loi.

Musée zoologique de
l'Université de Moscou.

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le 25. VI. 1945.

* Nous n'avons pas vu personnellement la race *dolgushini* habitant les régions d'aval de la rivière Ili. On décrit cette race en disant que «le ton général de la fourrure sur le dos de l'animal est gris très clair avec un chatoiement roux». Cette description est trop incomplète pour permettre de ranger cette race dans l'un de deux groupes considérés; mais vraisemblablement il se rapporte au premier.